The FDS Pressure Equation

Intuitive Understanding and Solution Strategies

Dr. Susanne Kilian
hhpberlin - Ingenieure für Brandschutz
10245 Berlin - Germany

Agenda

The FDS pressure equation

Design principles of different pressure solvers

Presentation of different pressure solvers

The FDS Pressure Equation

$$\nabla^2 H = -\left[\frac{\partial(\nabla \cdot \mathbf{u})}{\partial t} + \nabla \cdot \mathbf{F}\right]$$

Changes of velocity field with time due to thermodynamic quantities

Influence of gravity, particle drag, viscosity, perturbation pressure

The FDS Pressure Equation

$$\nabla^2 H = - \underbrace{\left[\frac{\partial (\nabla \cdot \mathbf{u})}{\partial t} + \nabla \cdot \mathbf{F} \right]}_{\downarrow \; =: \; R}$$

Right hand side abbreviated, computed by FDS

Elliptic Poisson equation

Bernoulli pressure H

$$H \equiv |\mathbf{u}|^2/2 + \tilde{p}/\rho$$

only perturbation pressure

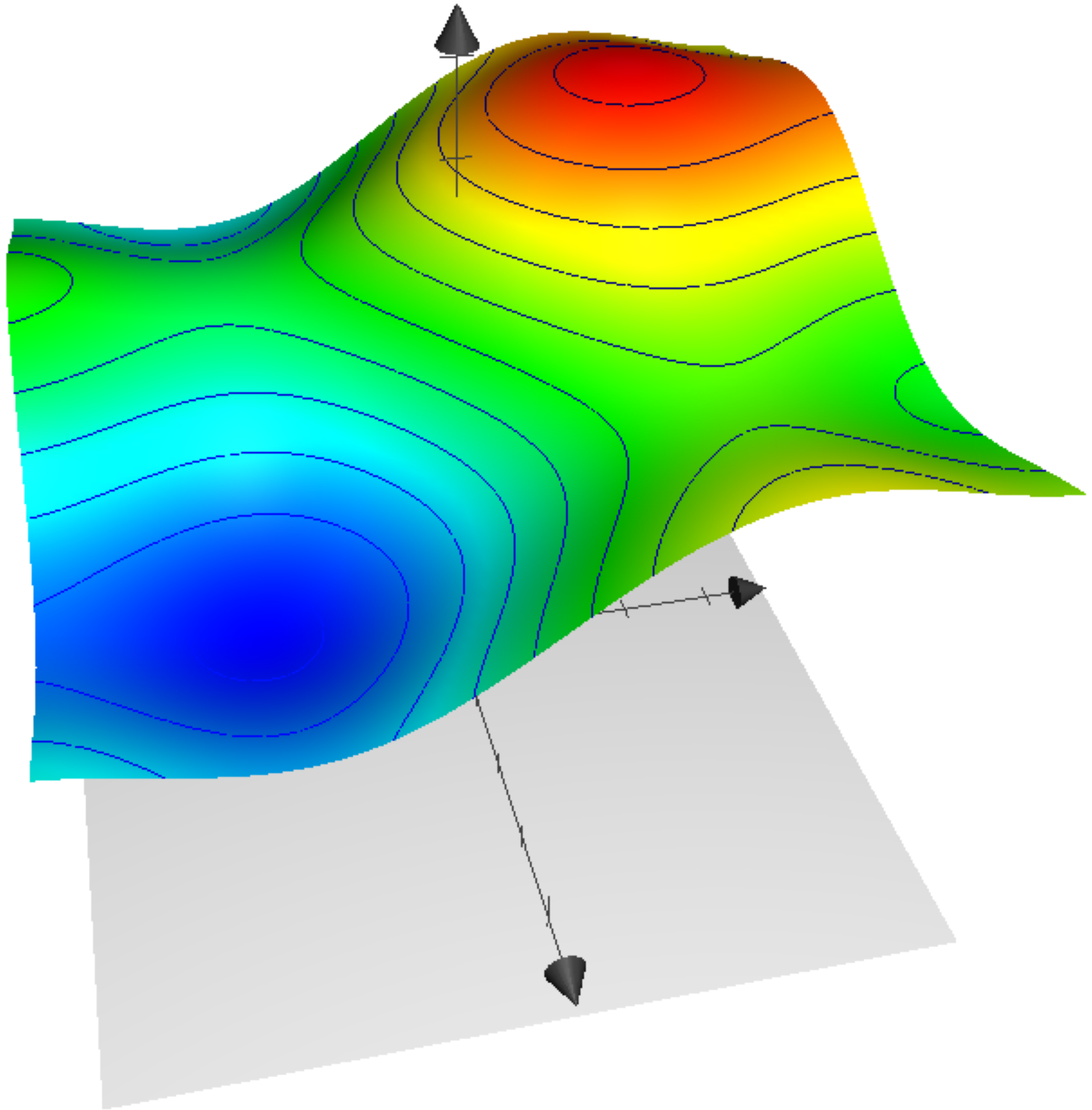
$$-\nabla^2 H = R$$

$$= \mathrm{div}\,(\mathrm{grad}\, H)$$

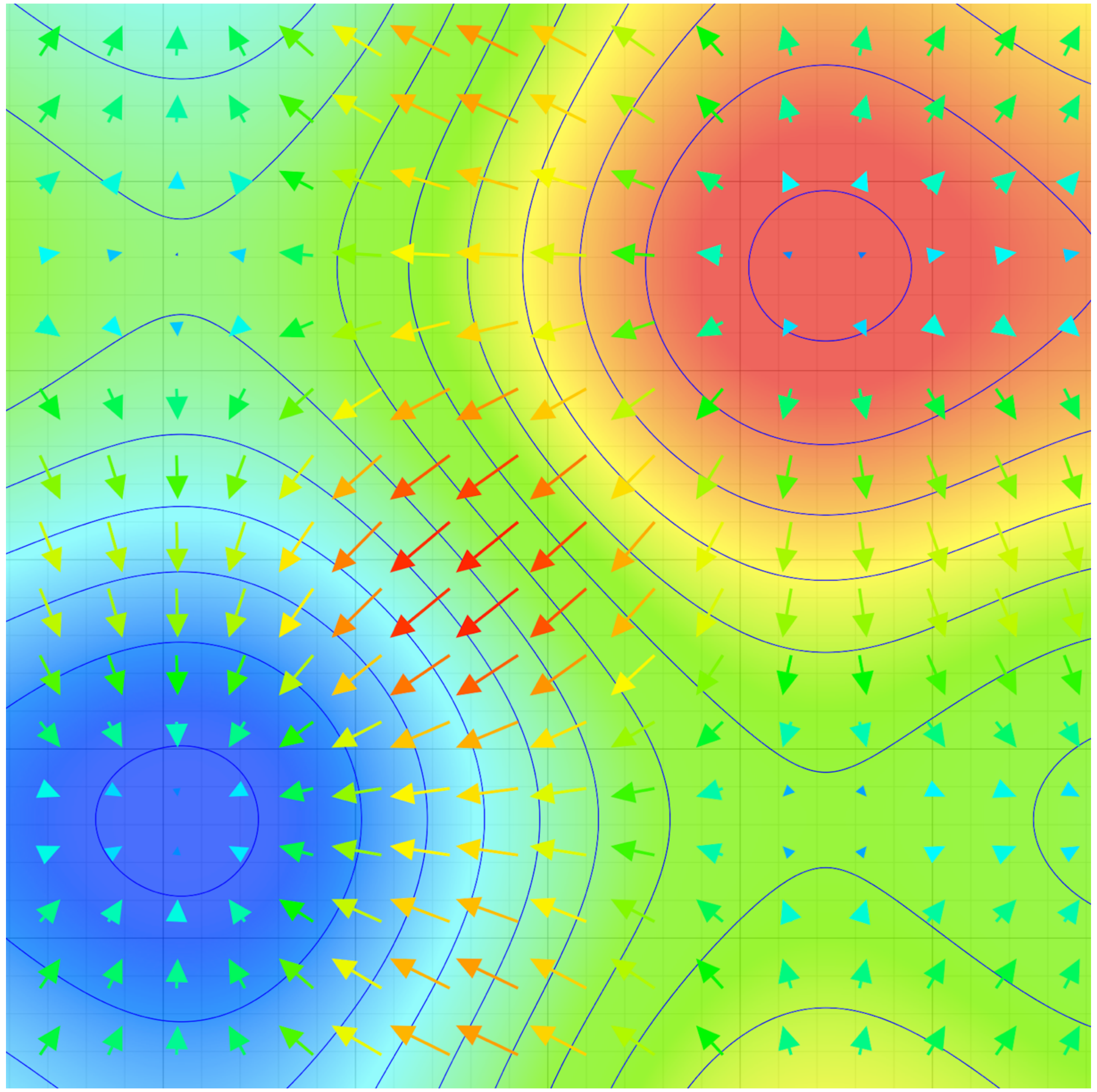
Divergence of pressure gradient field

Divergence of pressure gradient field

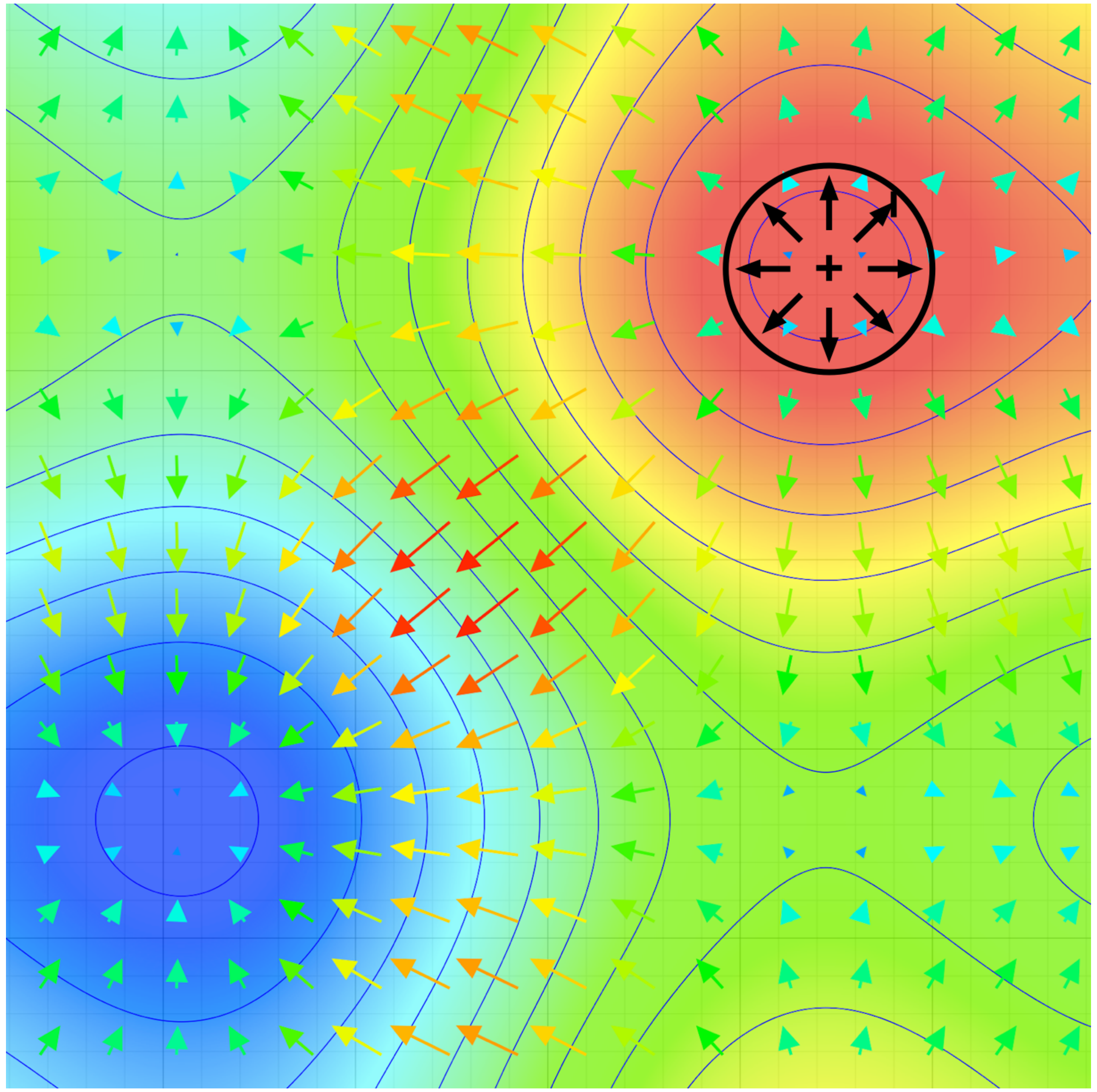
2D-Example pressure field H(x,y)



Negative gradient $-\nabla H$



Divergence of pressure gradient field



Positive divergence

Vectors are oriented outwards in all directions,

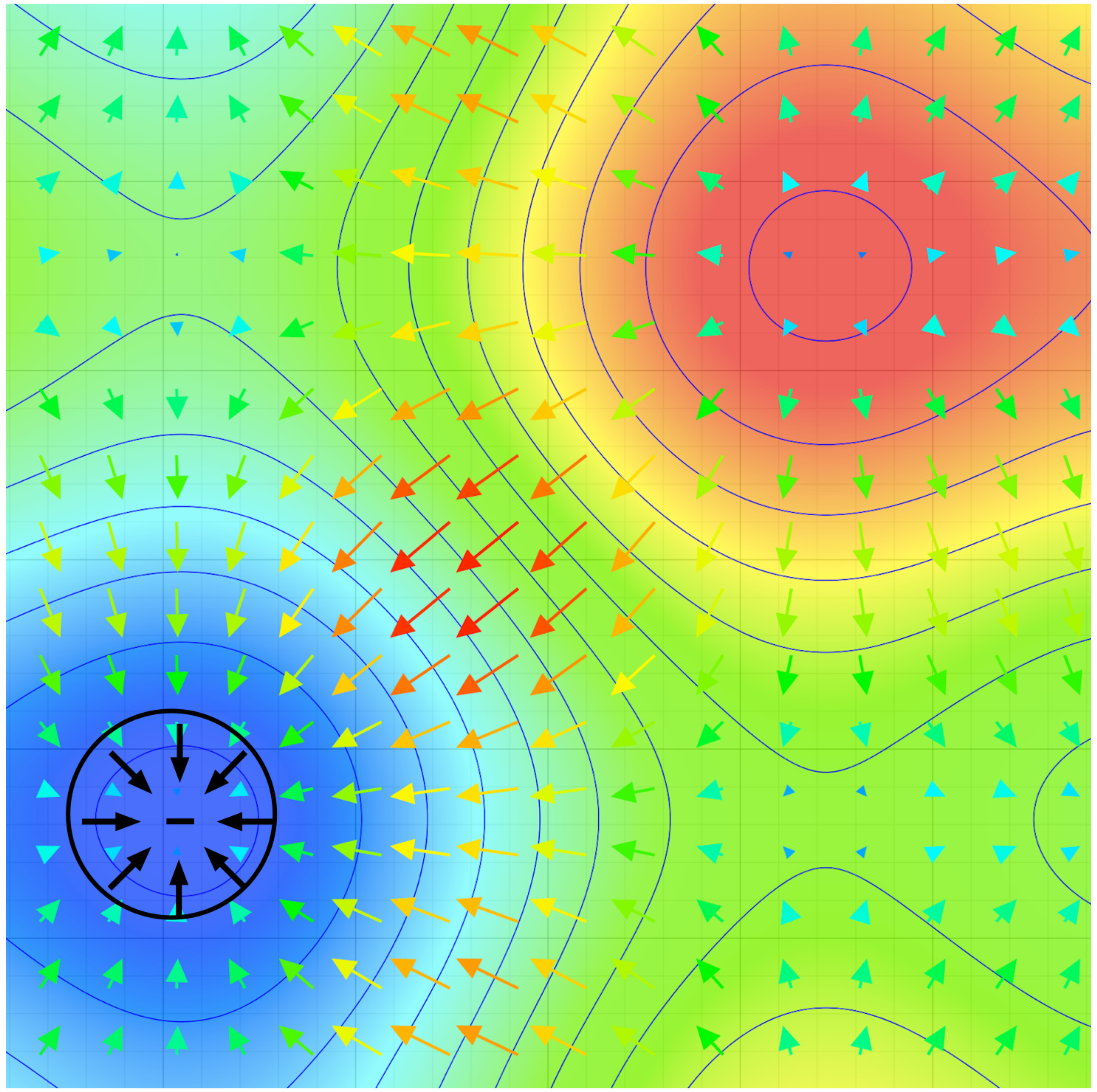
flow diverges (kind of source)

Divergence of pressure gradient field

Negative divergence

Vectors are oriented inwards from all directions,

flow converges (kind of sink)

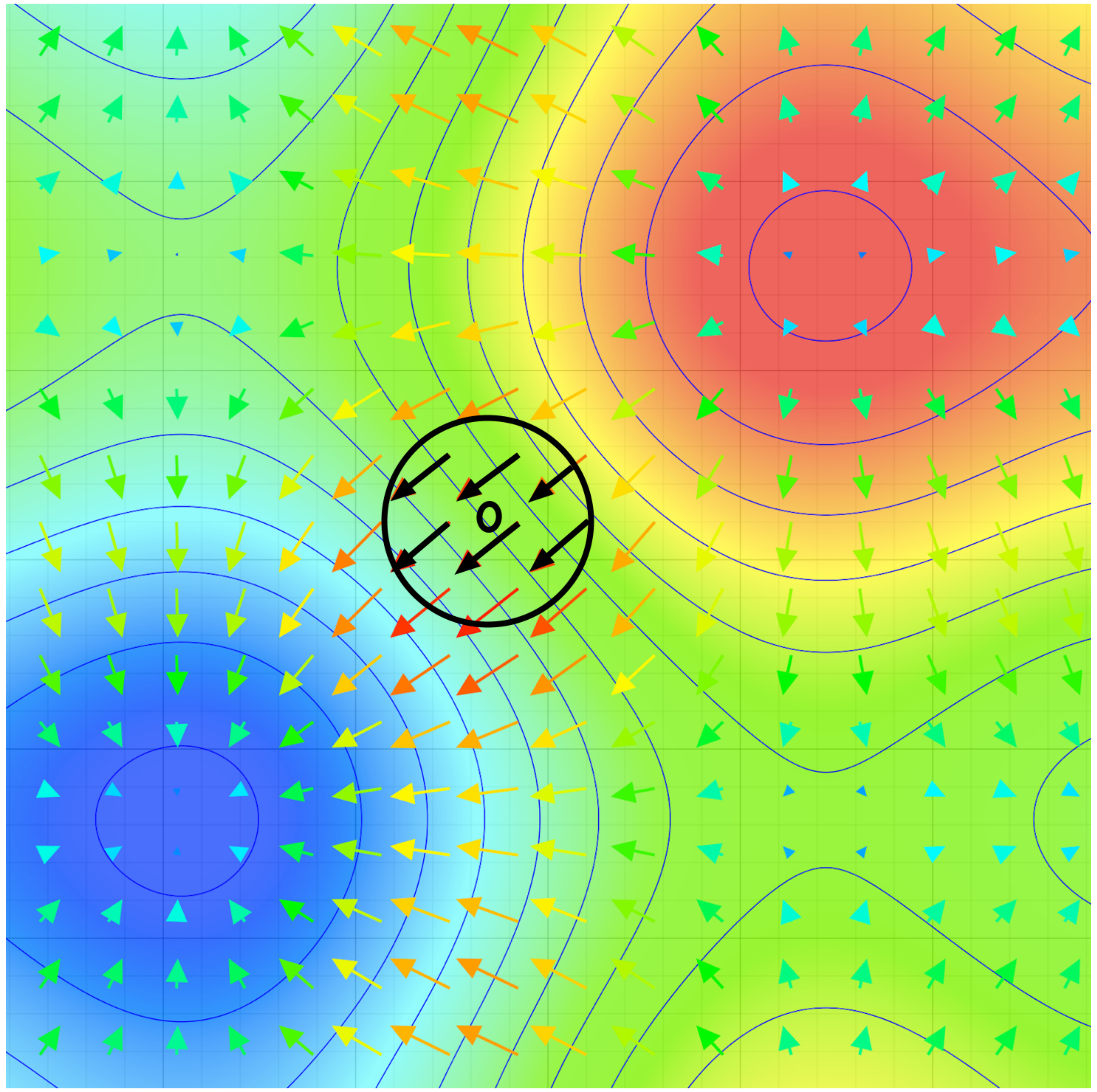


Divergence of pressure gradient field

Zero divergence

In- and outgoing vectors are evenly balanced

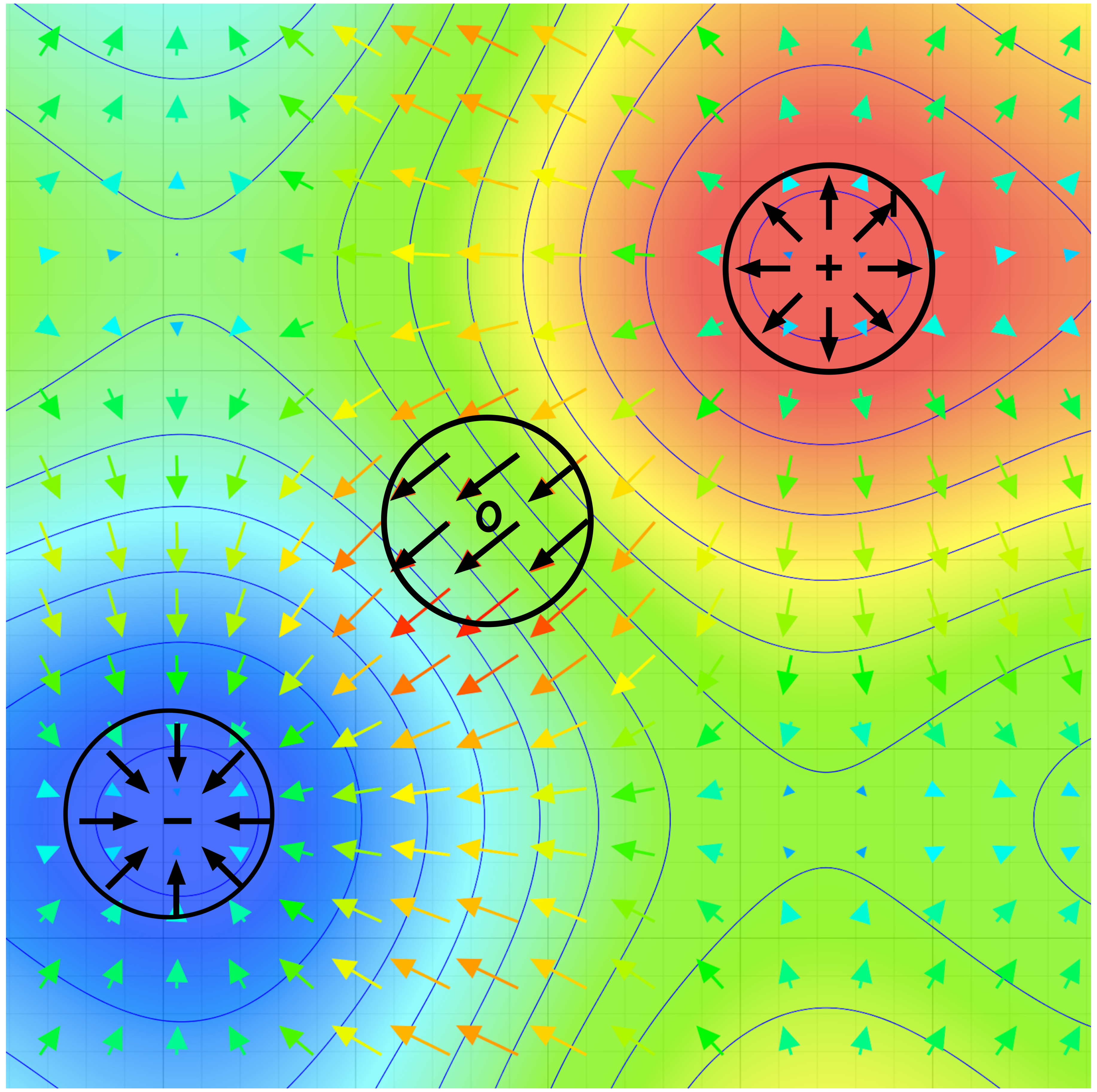
(kind of balance)



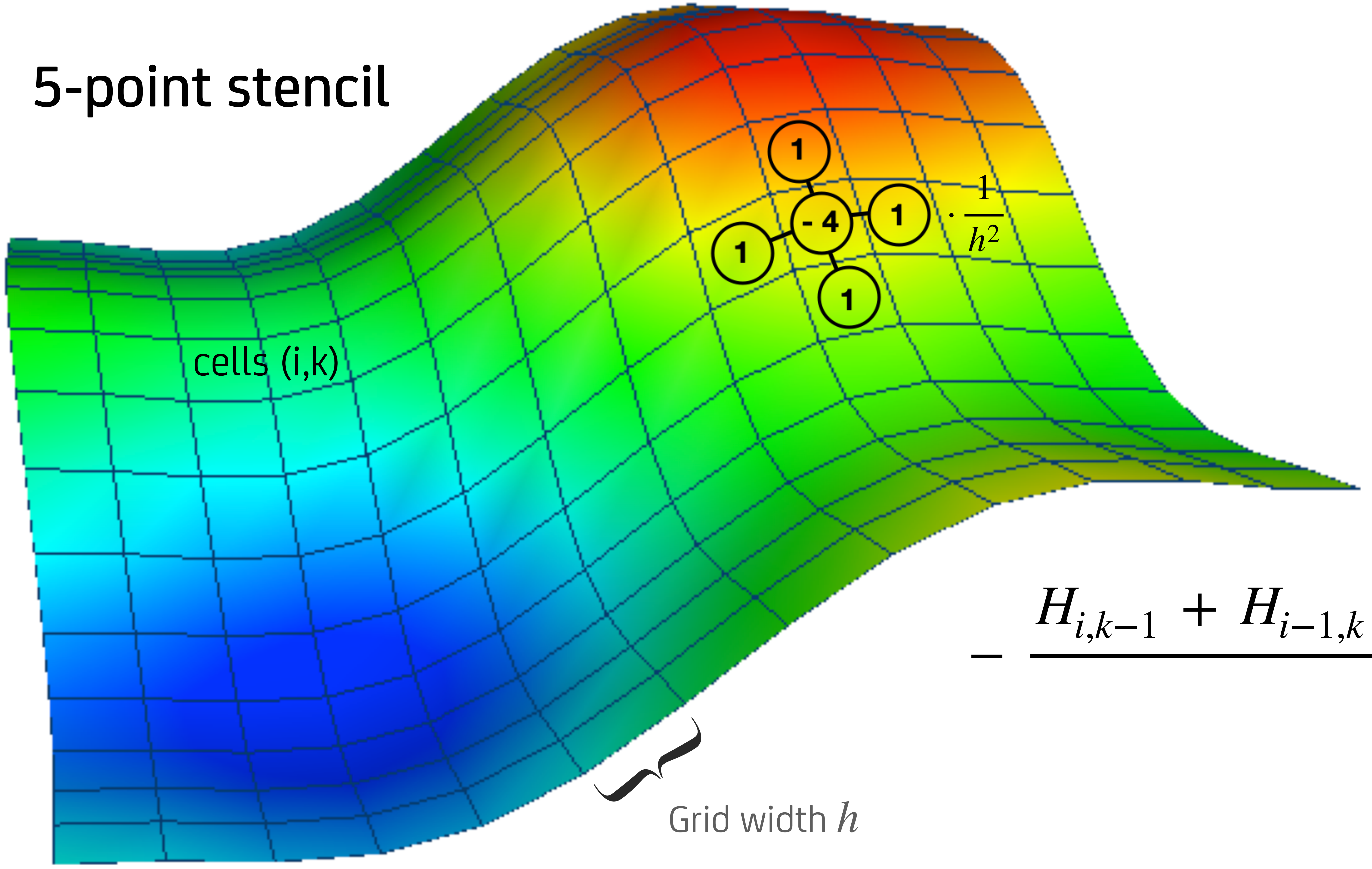
Divergence of pressure gradient field

Local measure of how much

the flow field expands/contracts



Discretization by Finite Differences



$$-\nabla^2 H = R$$

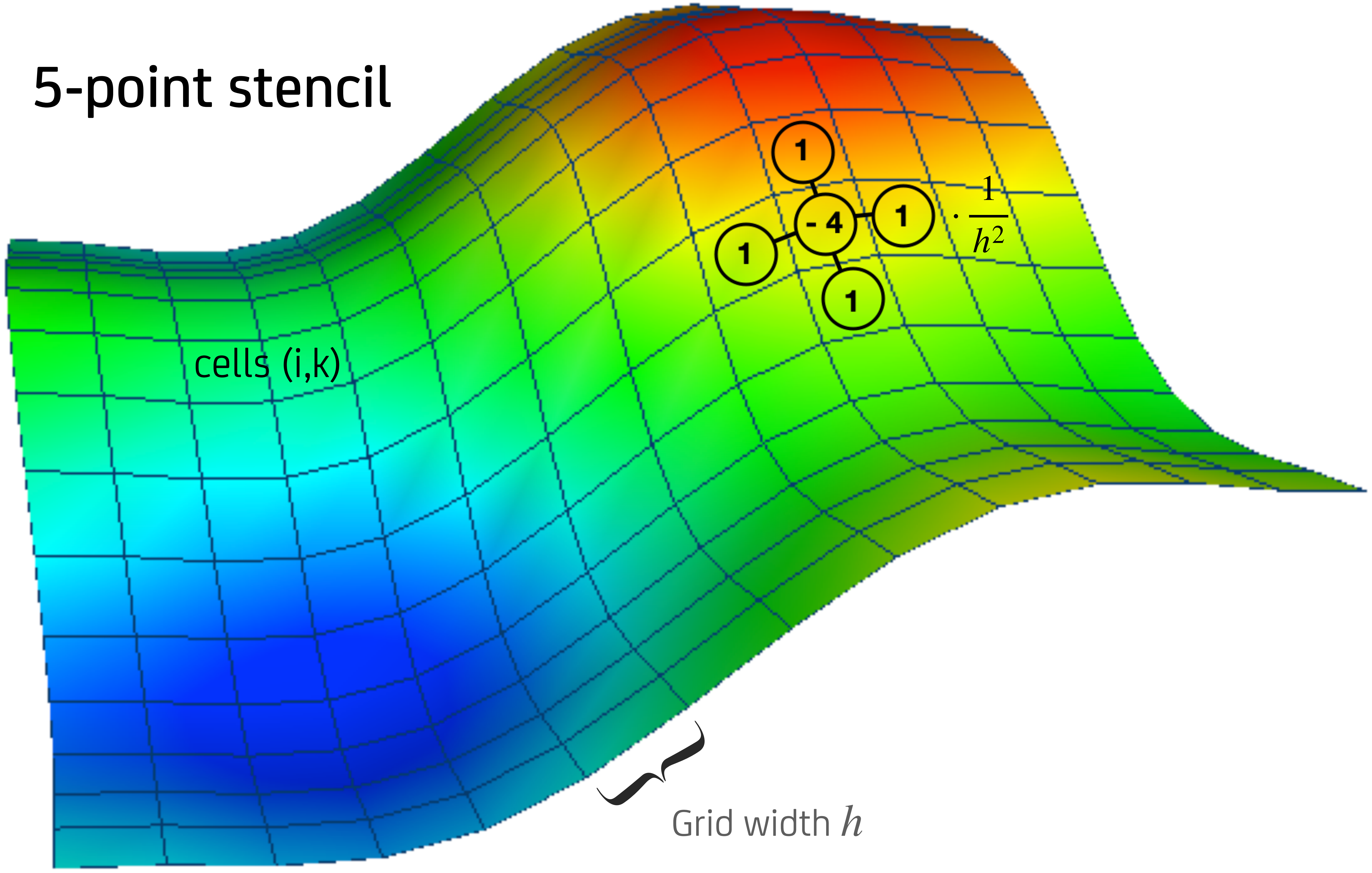
Values in cell centers

$$-\frac{H_{i,k-1} + H_{i-1,k} - 4\,H_{i,k} + H_{i+1,k} + H_{i,k+1}}{h^2} = R_{i,k}$$

+ Boundary conditions

Discretization by Finite Differences

5-point stencil



$$-\nabla^2 H = R$$

Analytical equation

$$Ax = b$$

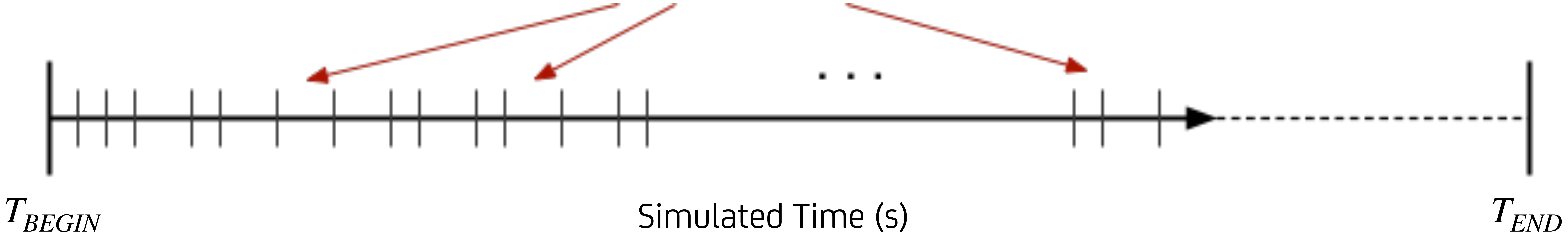
Algebraic system

Numerical solution

To be solved at least twice in every time step

Efficiency & accuracy
very important!

$$Ax = b$$



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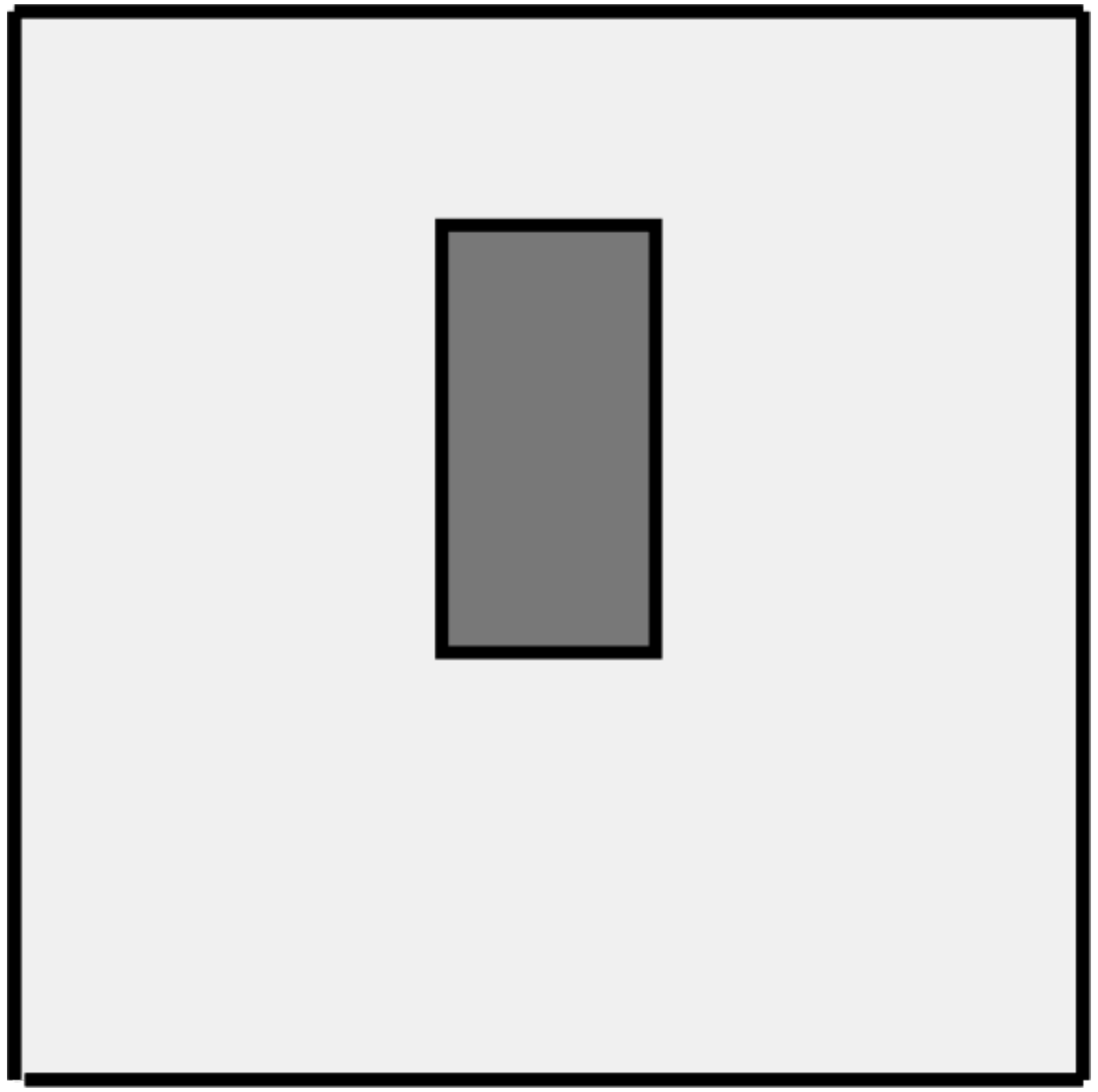
Design principles of different pressure solvers

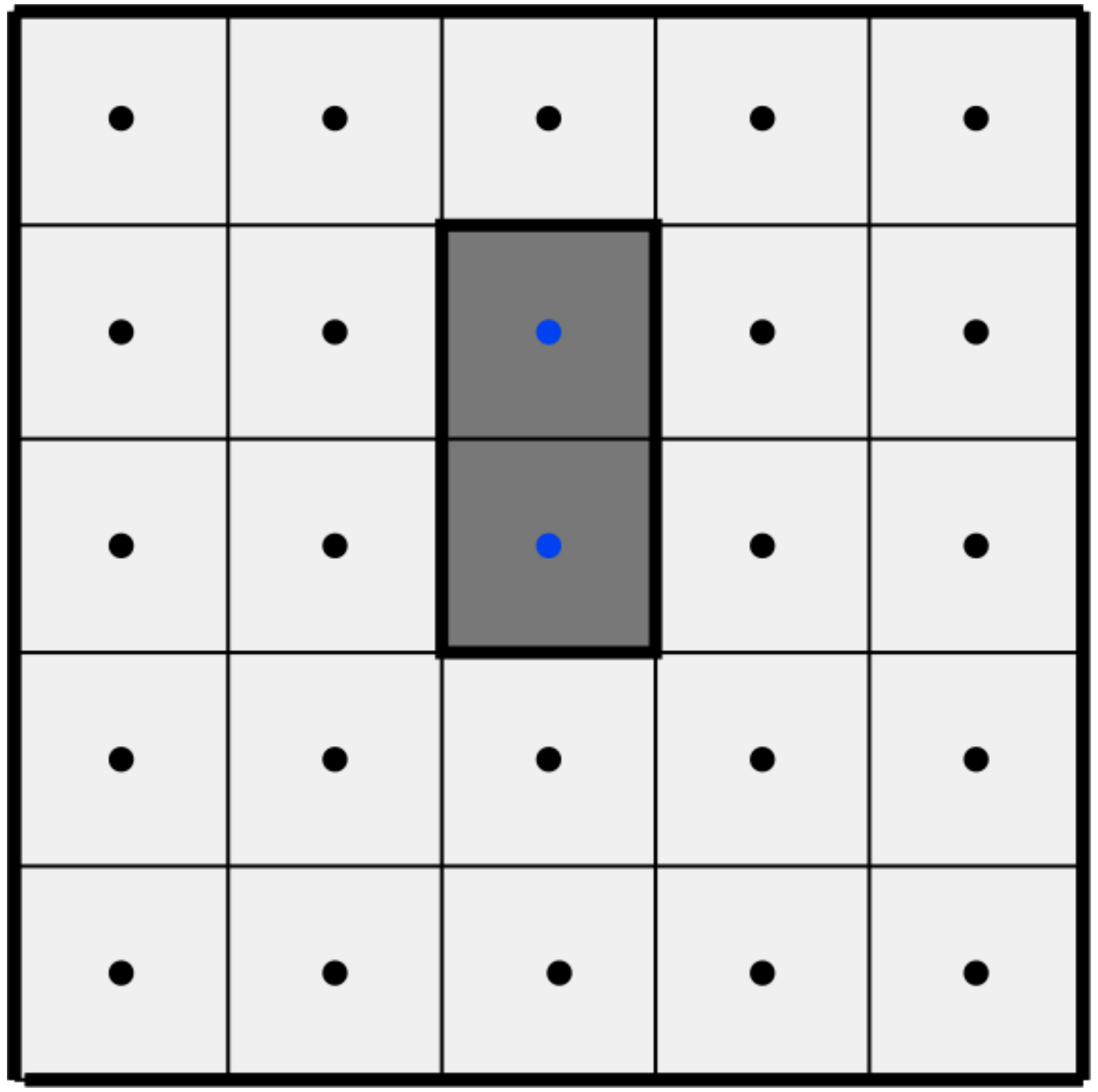
Presentation of different pressure solvers

Discretization types

Rectangular mesh with obstruction



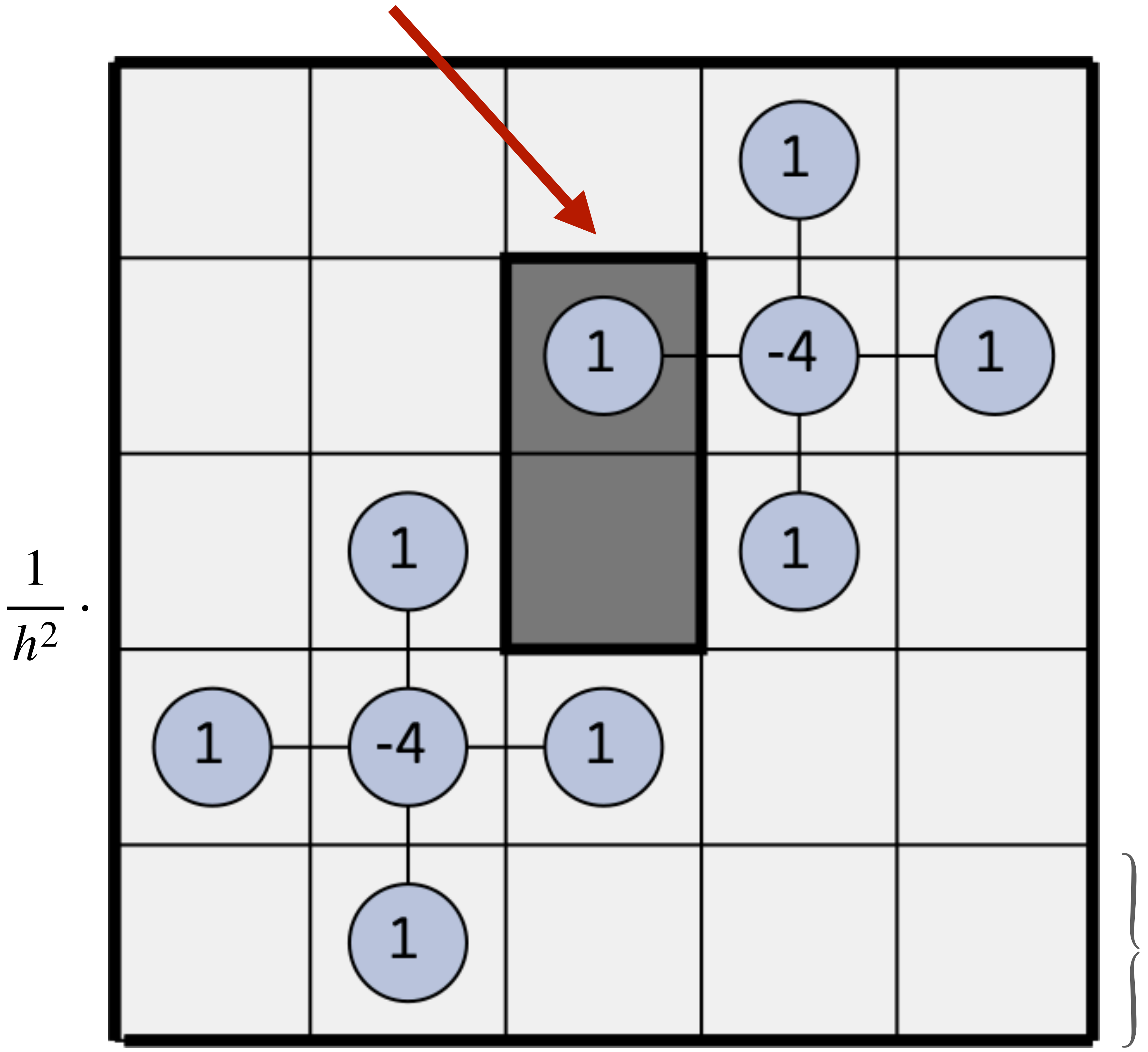
Cell-centered values of H



Discretization types

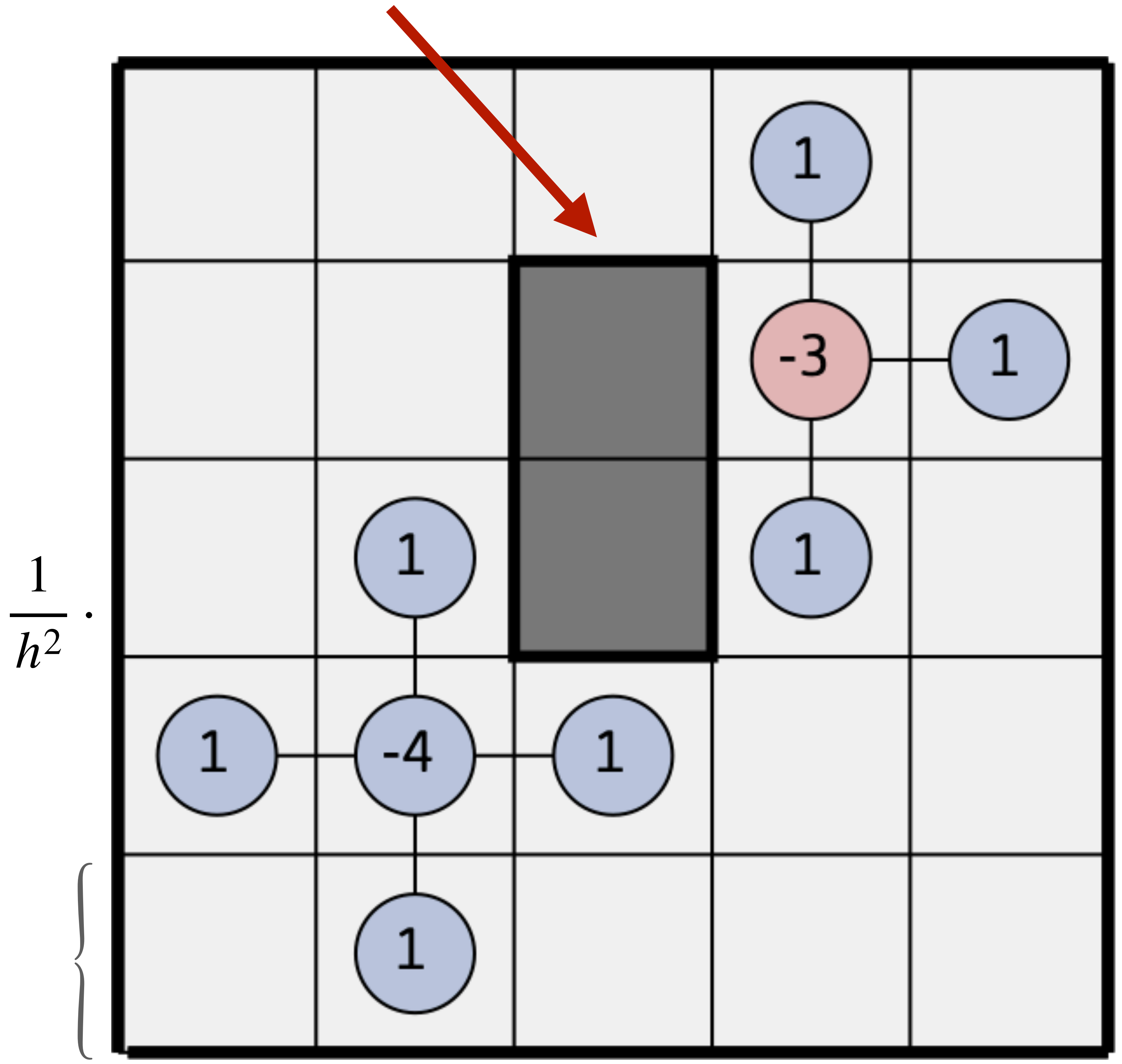
Structured

Obstructions included, part of matrix A



Unstructured

Obstructions excluded, NOT part of matrix A

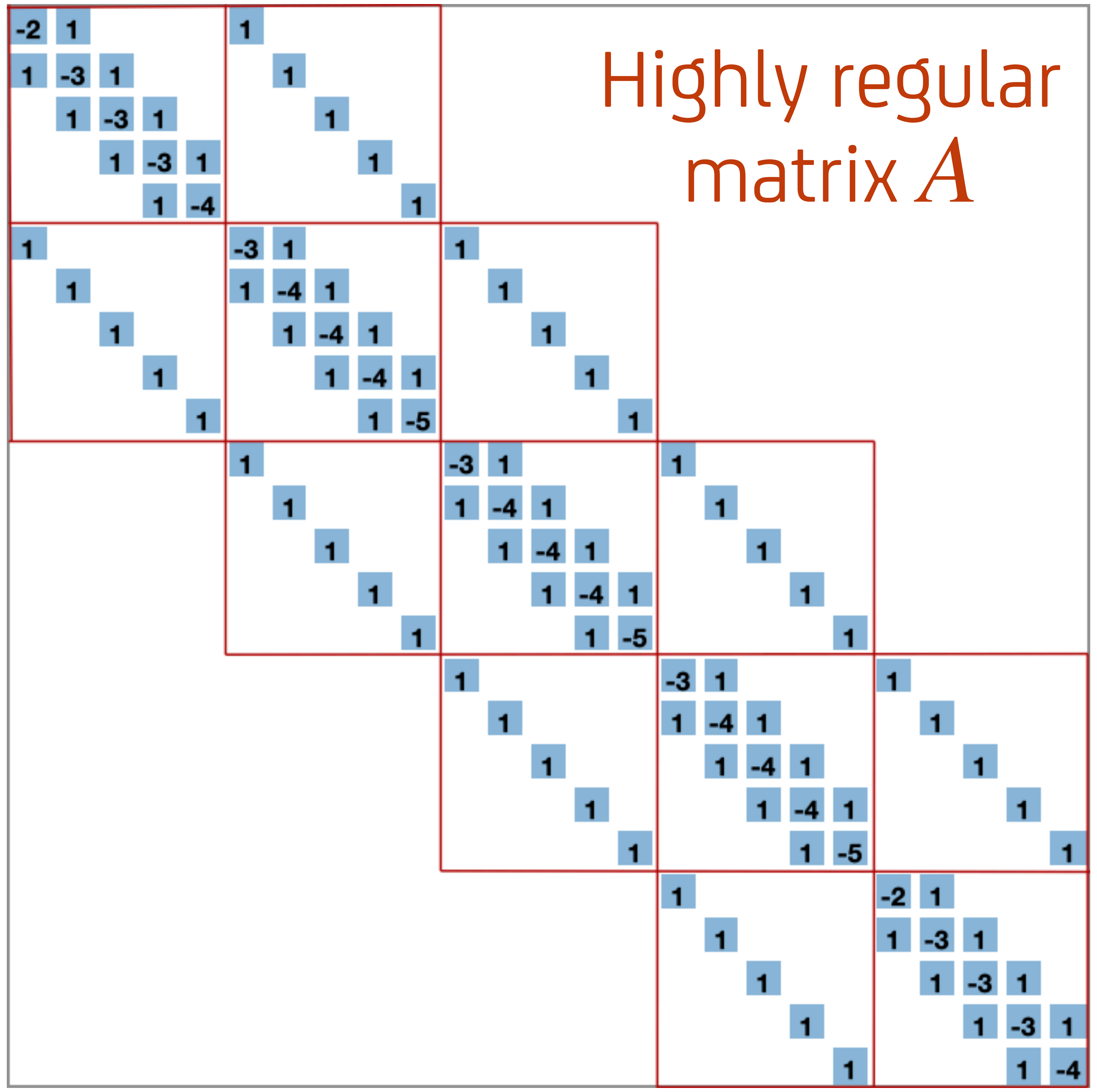


Grid width h

Discretization types

Structured

Very efficient solvers,
but not correct at obstructions



$$\frac{1}{h^2}\cdot\left(\begin{array}{ccccccccccccccccccccccccc}
-2 & 1 & & & & 1 \\
1 & -3 & 1 & & & & 1 \\
 & 1 & -3 & 1 & & & & 1 \\
 & & 1 & -3 & 1 & & & & 1 \\
 & & & 1 & -4 & & & & & 1 \\
1 & & & & & -3 & 1 & & & & 1 \\
 & 1 & & & & 1 & -4 & 1 & & & & 1 \\
 & & 1 & & & & 1 & -4 & 1 & & & & 1 \\
 & & & 1 & & & & 1 & -4 & 1 & & & & 1 \\
 & & & & 1 & & & & 1 & -5 & & & & & 1 \\
 & & & & & 1 & & & & & -3 & 1 & & & & 1 \\
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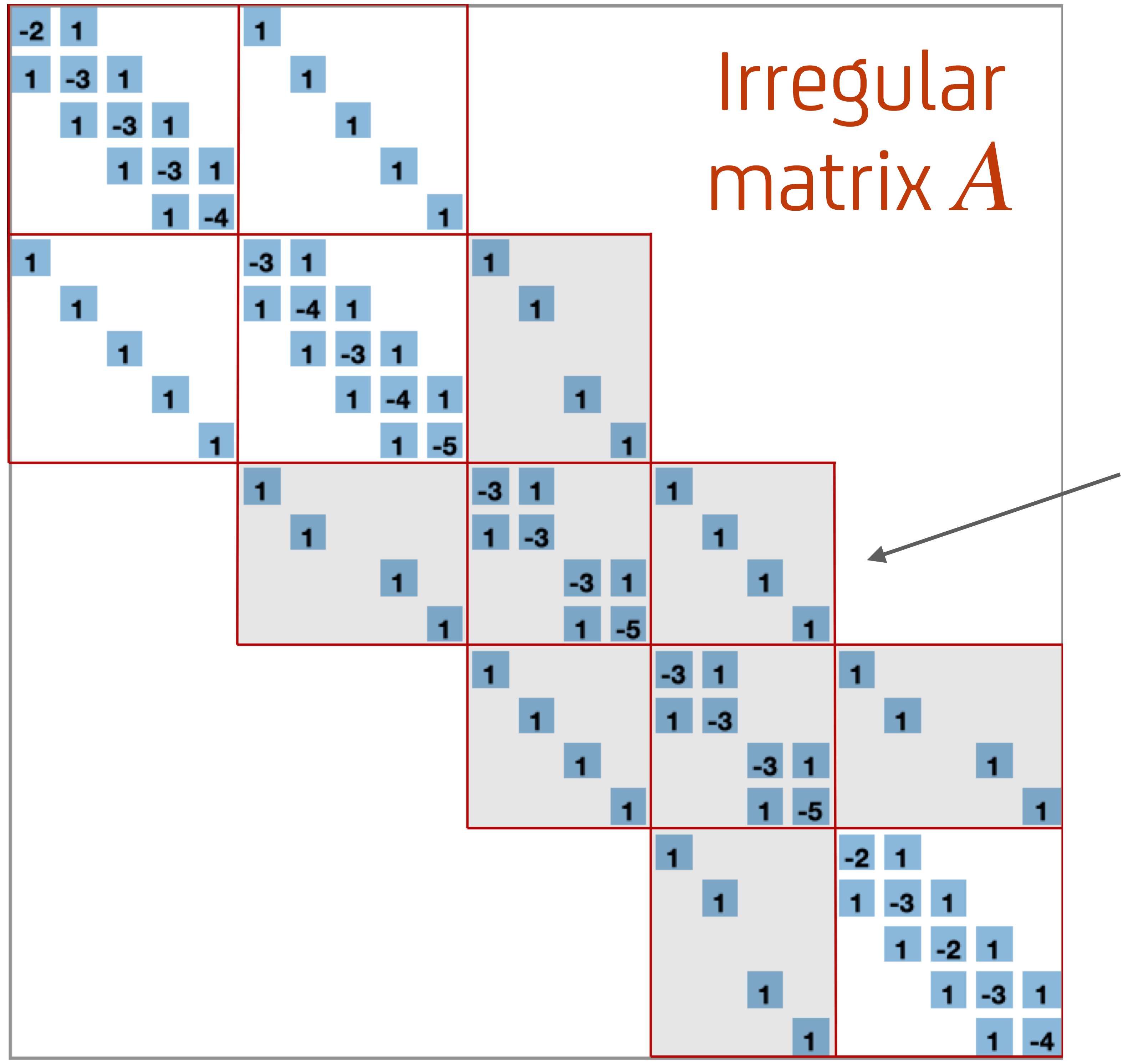
 & & & & & & & & & & & & & & & & & & 1 & & & & 1 & -3 & 1 \\
 & & & & & & & & & & & & & & & & & & & 1 & & & & 1 & -4
\end{array}\right)$$

Highly regular
matrix A

Sparsity:
many zero
entries

Unstructured

Less efficient solvers,
but correct at obstructions



$$\frac{1}{h^2}\cdot\left(\begin{array}{ccccccccccccccccccccccc}
-2 & 1 & & & & 1 \\
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 & & 1 & -3 & 1 & & & & 1 \\
 & & & 1 & -4 & & & & & 1 \\
1 & & & & & -3 & 1 & & & & 1 \\
 & 1 & & & & 1 & -4 & 1 & & & & 1 \\
 & & 1 & & & & 1 & -3 & 1 \\
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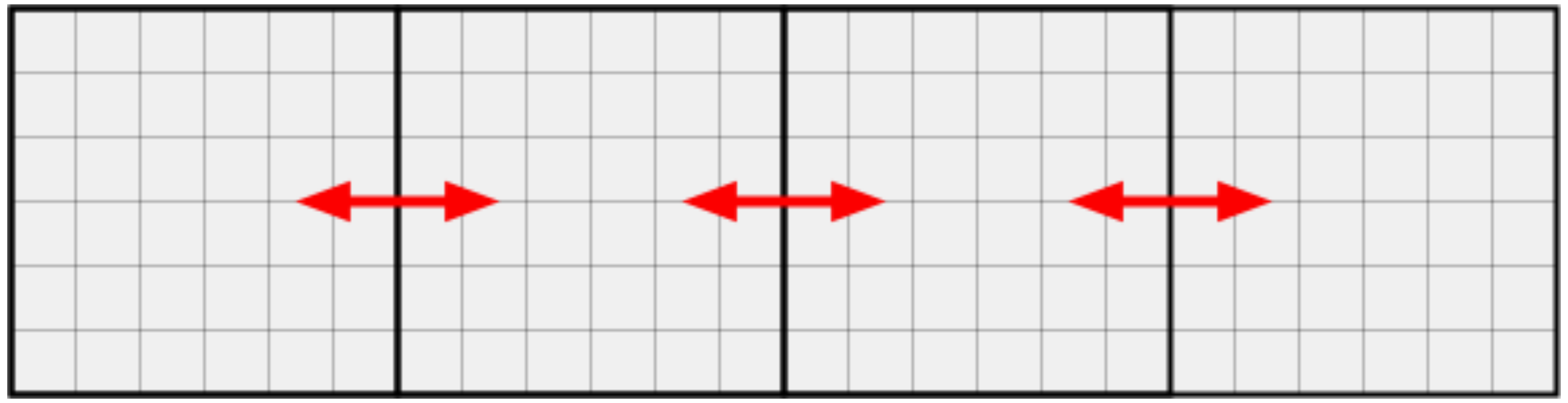
 & & & & & & & & & & & & & & & & & & & 1 & -2 & 1 \\
 & & & & & & & & & & & & & & & & 1 & & & & 1 & -3 & 1 \\
 & & & & & & & & & & & & & & & & & 1 & & & & 1 & -4
\end{array}\right)$$

Irregular
matrix A

less cells,
different
patterns

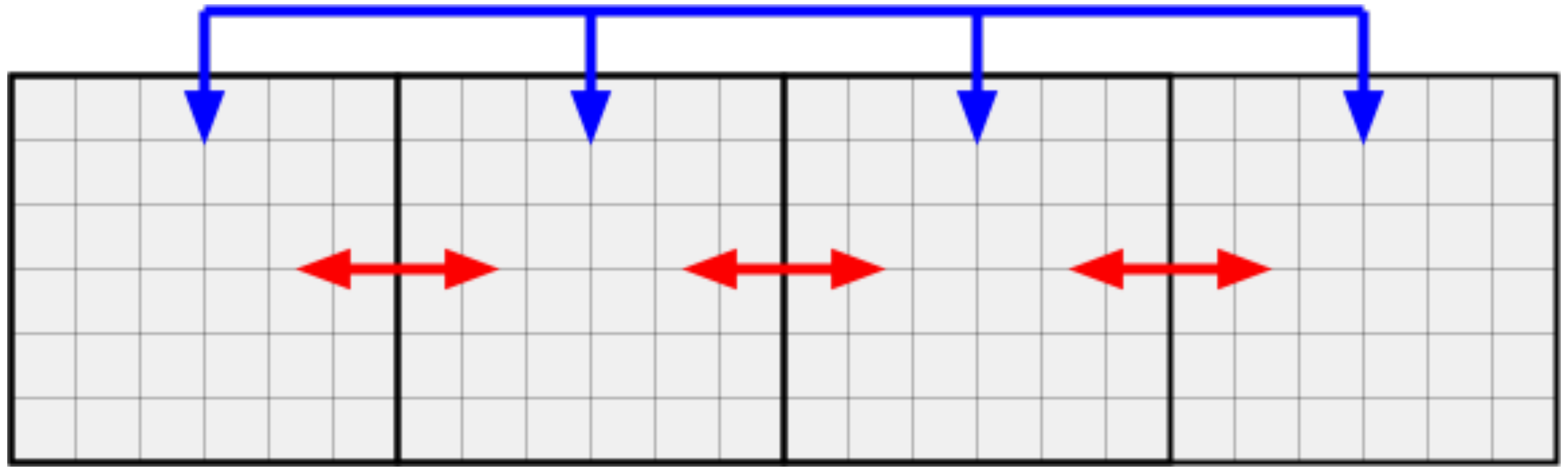
Data exchange types

Only Locally



Information is exchanged
only between neighbouring meshes

Locally and globally



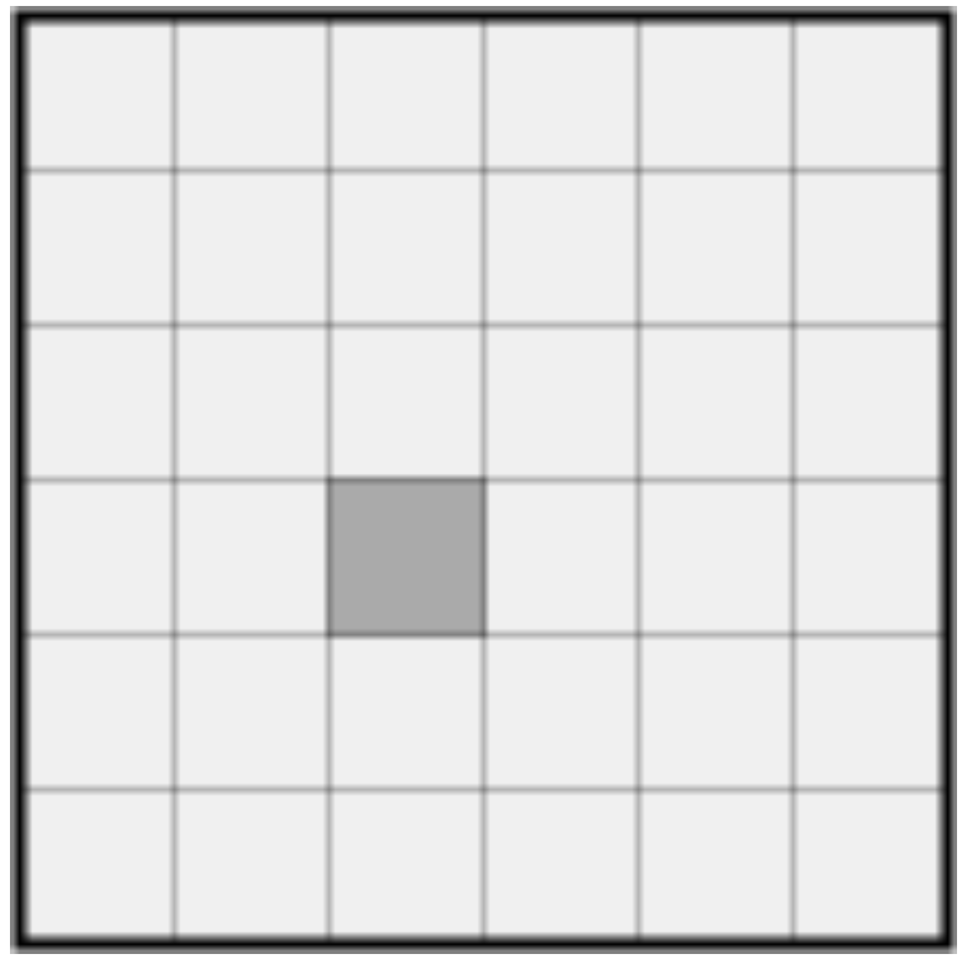
Information is exchanged
between all meshes

Different degrees of coupling of the whole solution

Highly coupled data in Poisson equation

New information (e.g. air blown in, fire ignited)

Single mesh case

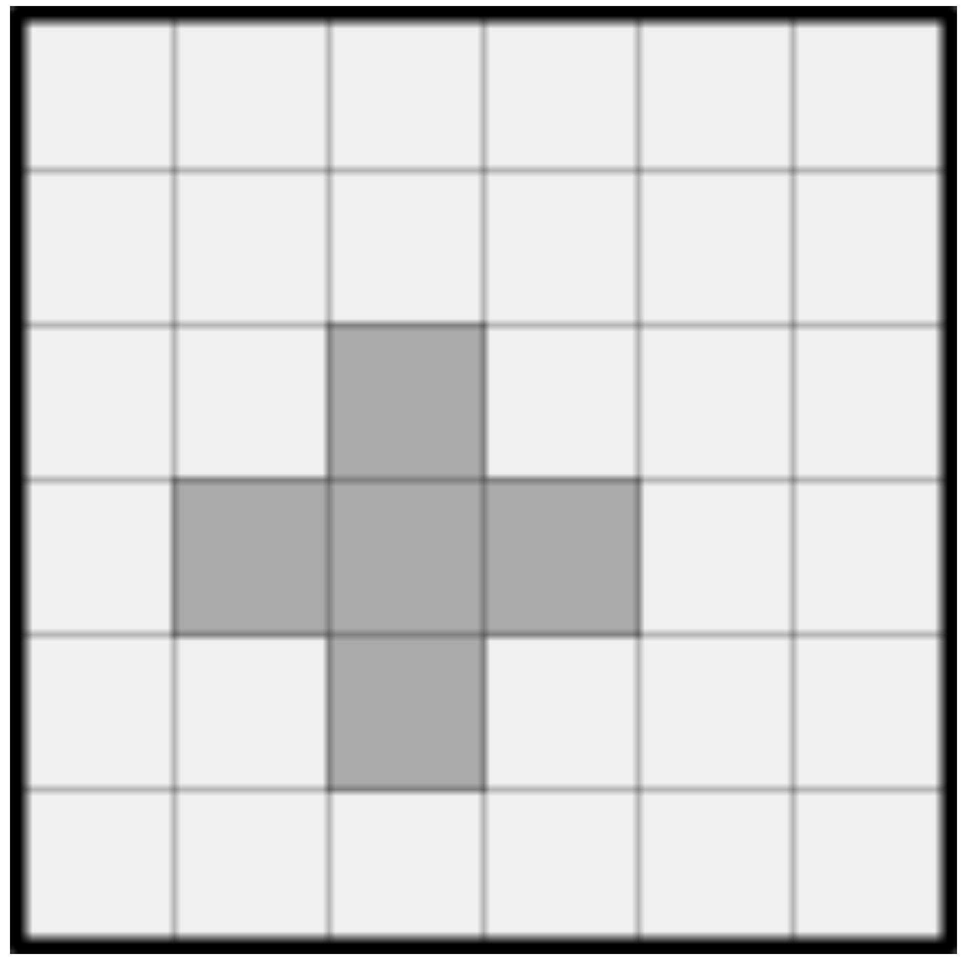


Cell values are defined as neighboring averages

Highly coupled data in Poisson equation

New information (e.g. air blown in, fire ignited)

Single mesh case



Cell values are defined as neighboring averages

Continuous chain of information, neighbor by neighbor

Highly coupled data in Poisson equation

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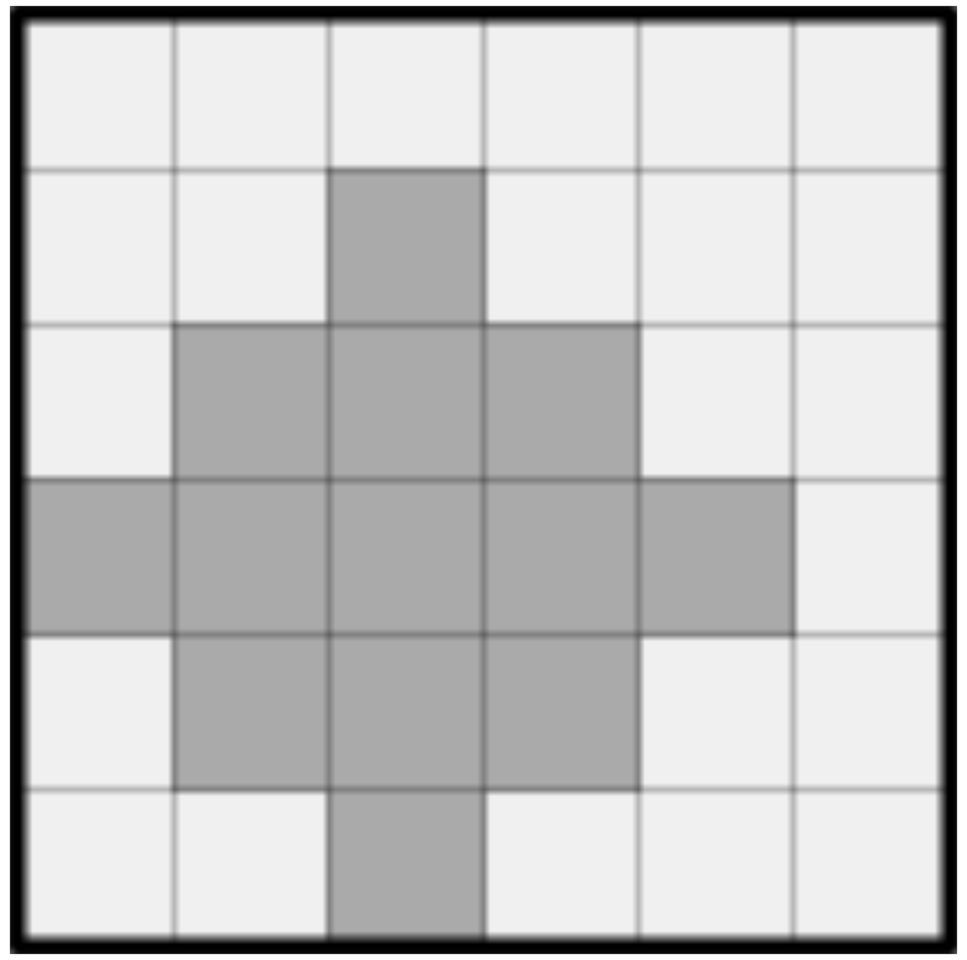
Single mesh case



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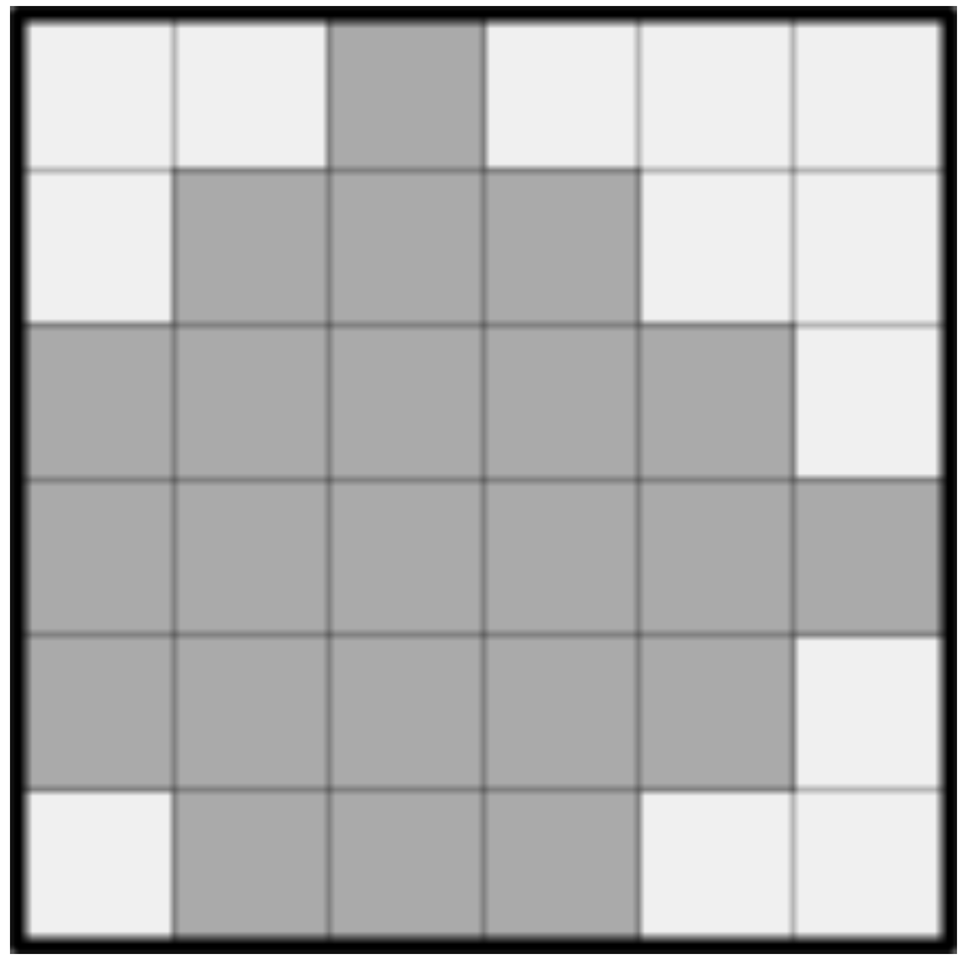

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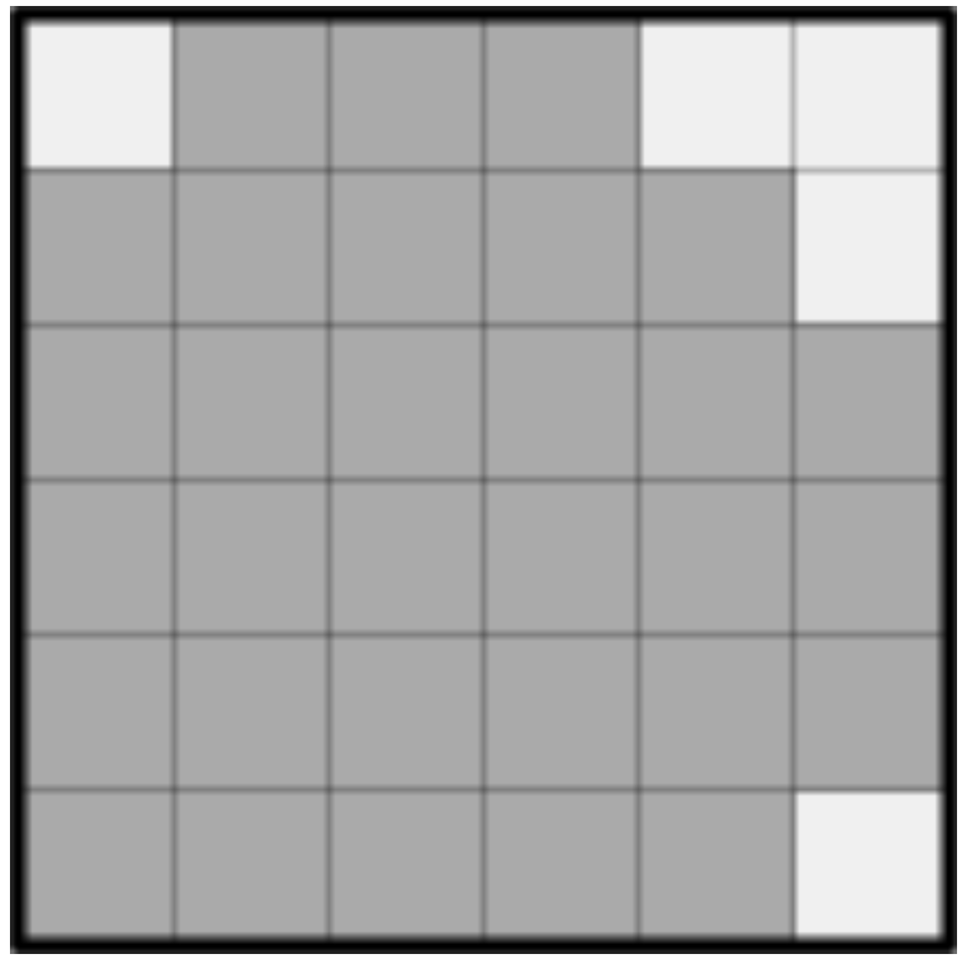

Cell values are defined as neighboring averages

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Highly coupled data in Poisson equation

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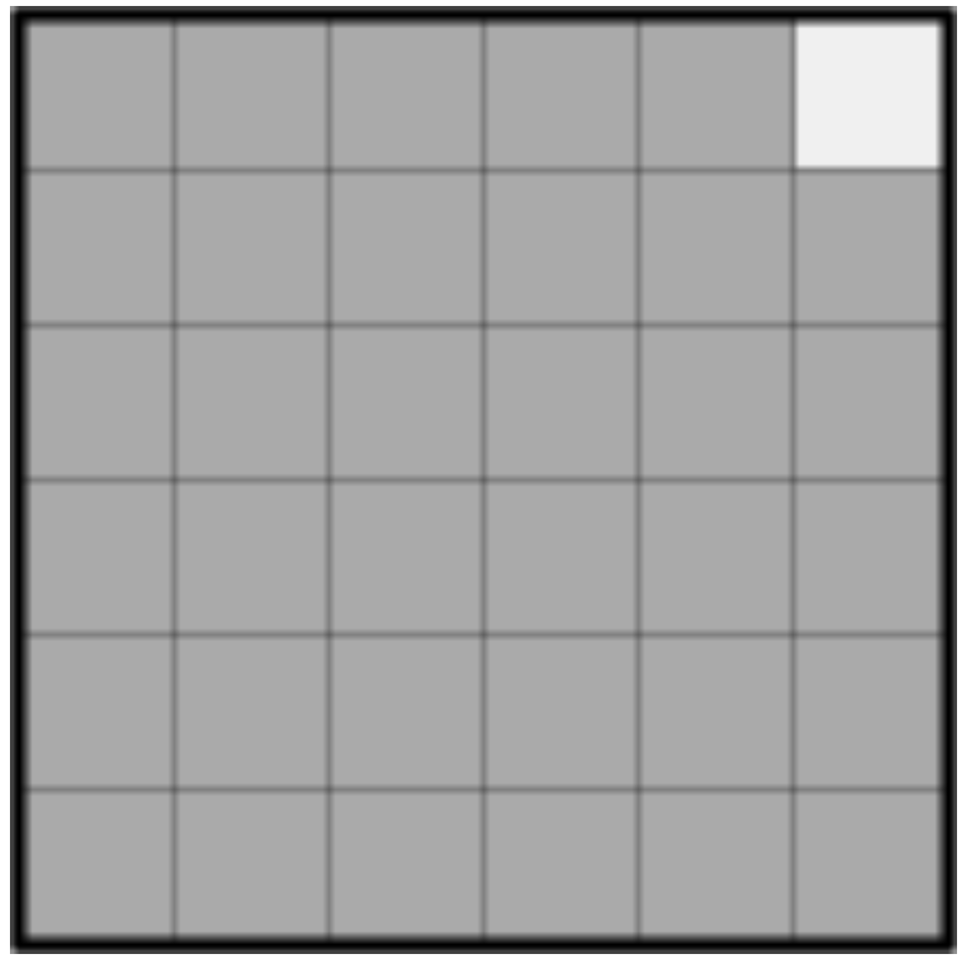
Single mesh case



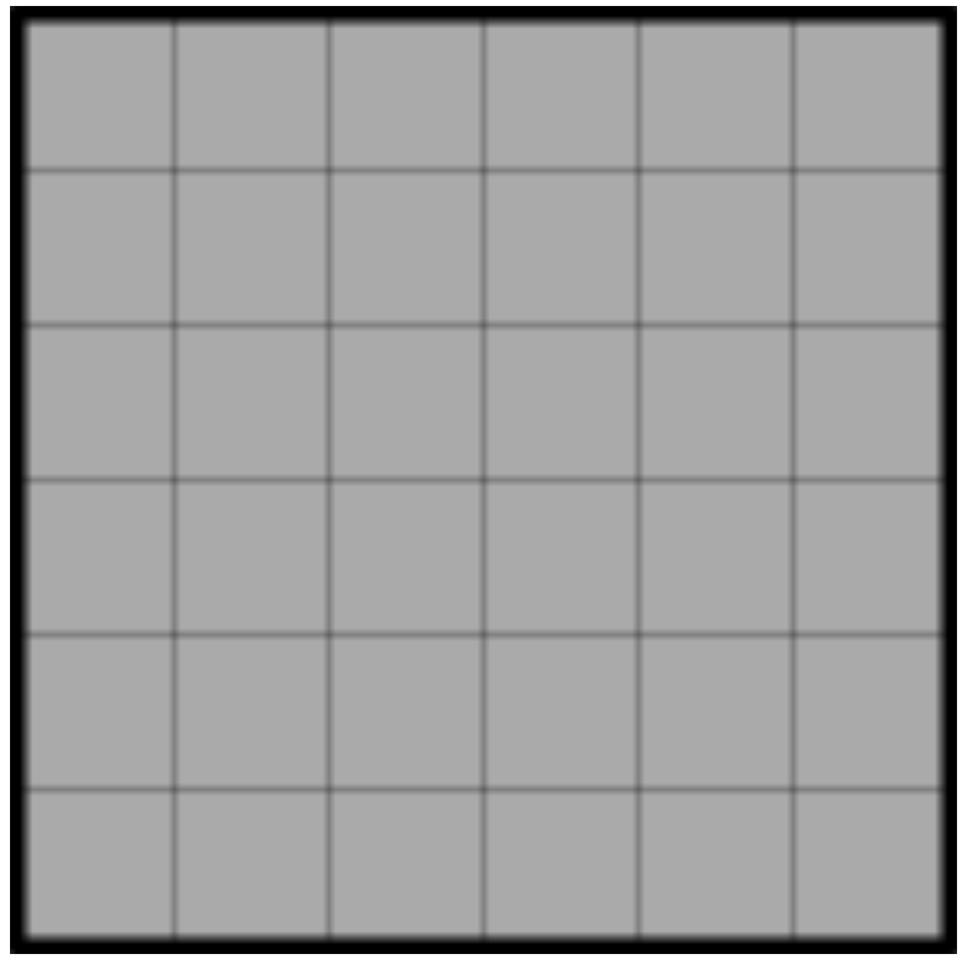
Cell values are defined as neighboring averages

Continuous chain of information, neighbor by neighbor

Highly coupled data in Poisson equation

New information (e.g. air blown in, fire ignited)

Single mesh case



Cell values are defined as neighboring averages

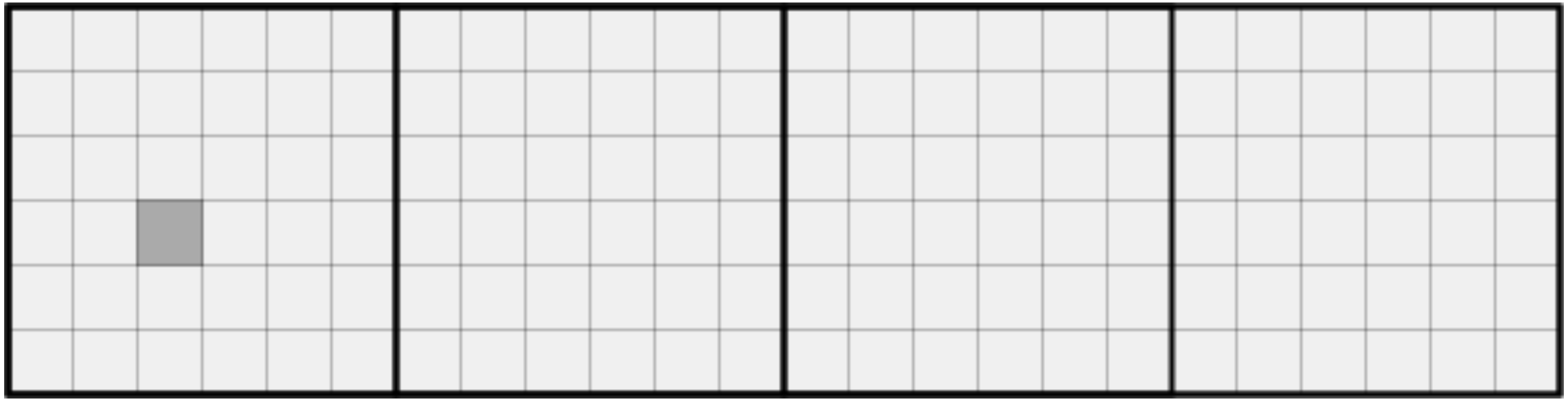
Continuous chain of information, neighbor by neighbor

Perceived by all cells in only ONE Poisson solution

Consequences for multi-mesh cases

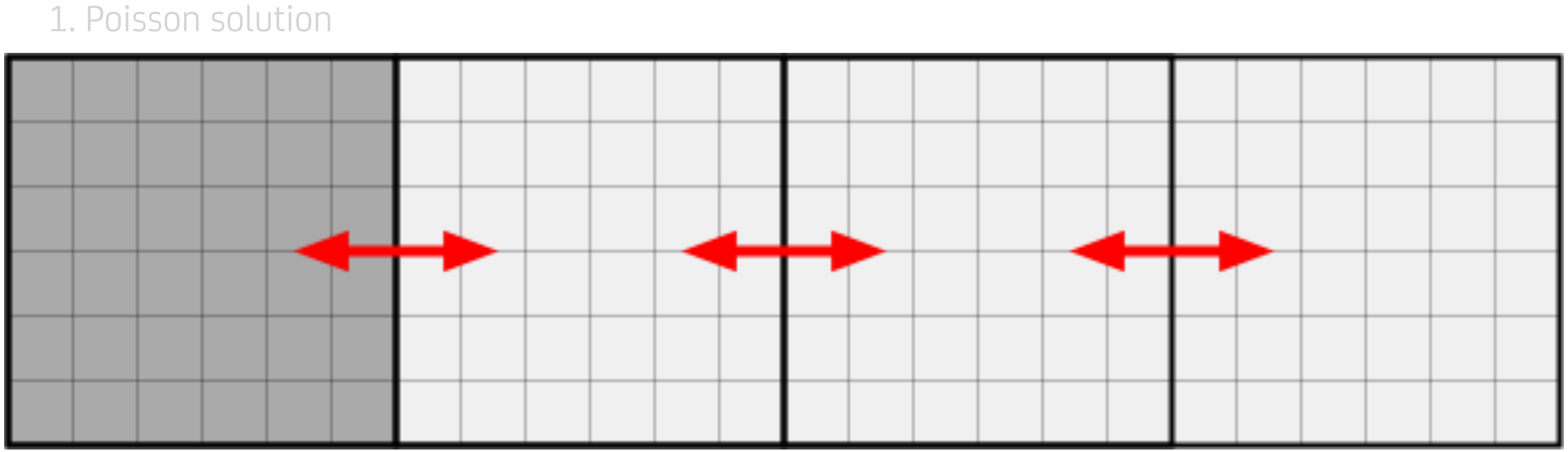
What happens for long domains subdivided into meshes?

Multi mesh case



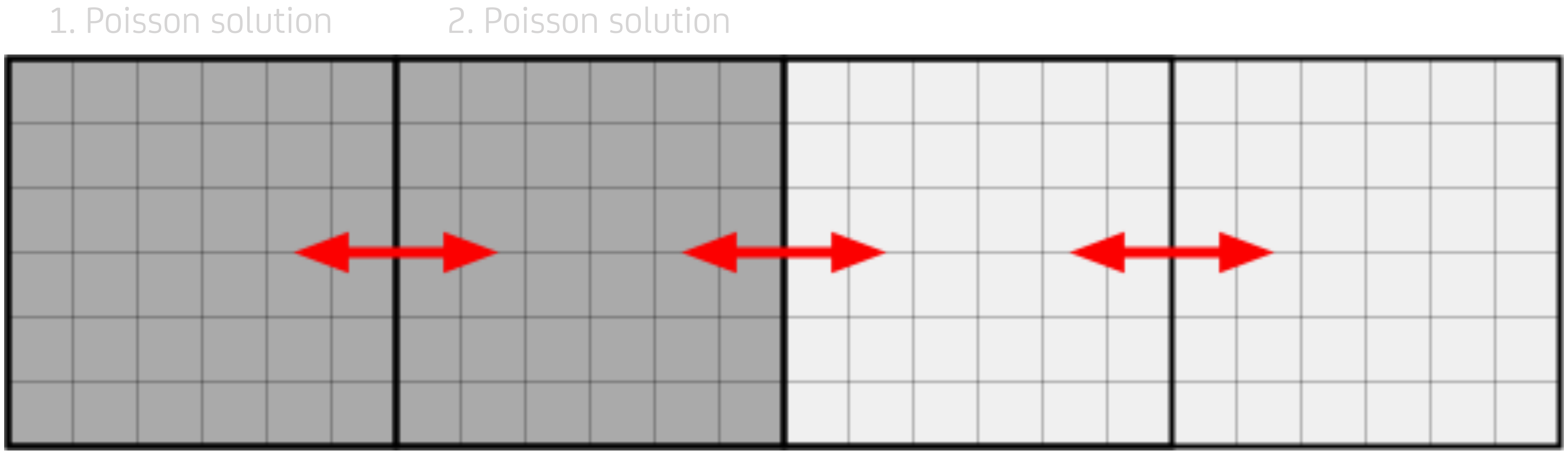
Consequences for multi-mesh cases

1. Pass: Information is first trapped in mesh 1



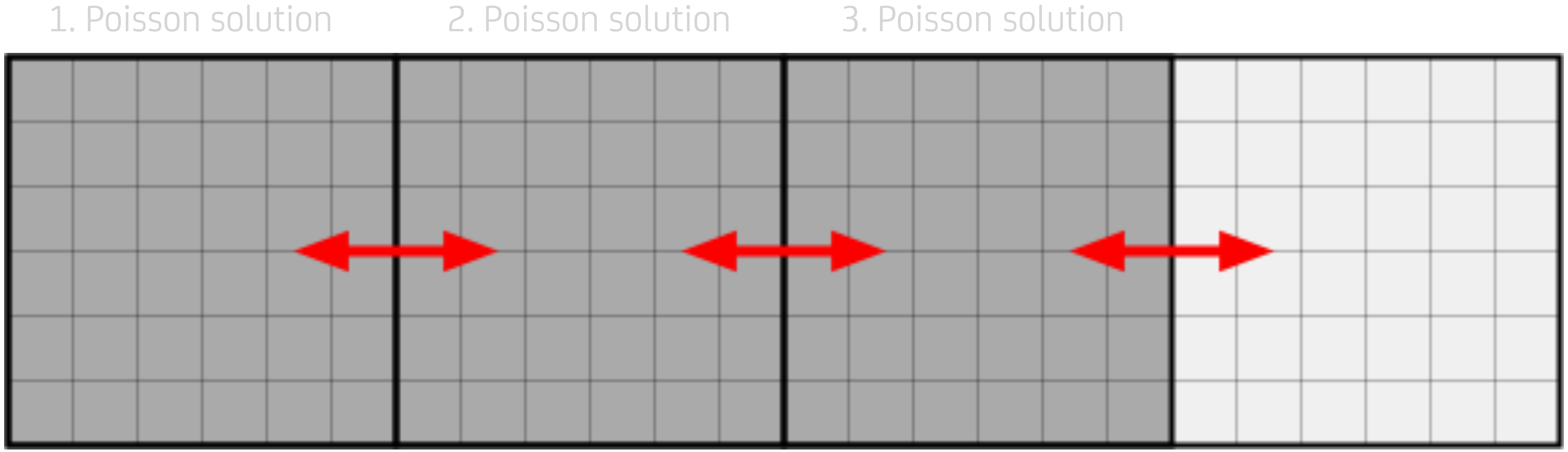
Consequences for multi-mesh cases

2. Pass: Information can only reach mesh 2



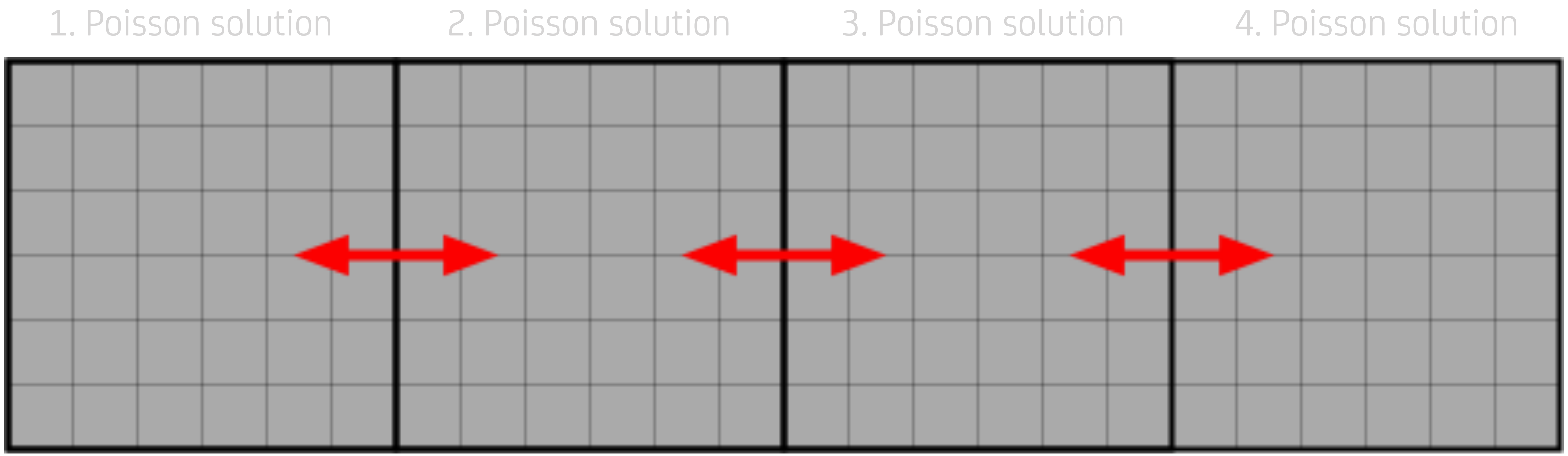
Consequences for multi-mesh cases

3. Pass: Information can only reach mesh 3



Consequences for multi-mesh cases

4. Pass: Information finally reaches mesh 4



Delay in information transfer

Agenda

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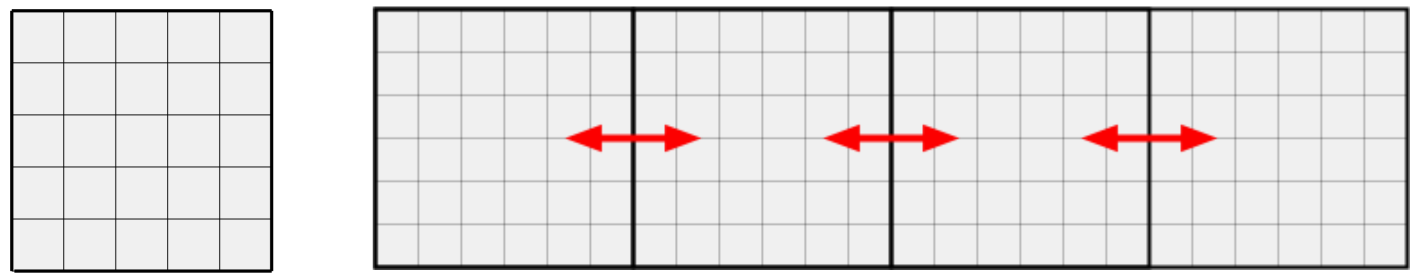
Design principles of different pressure solvers

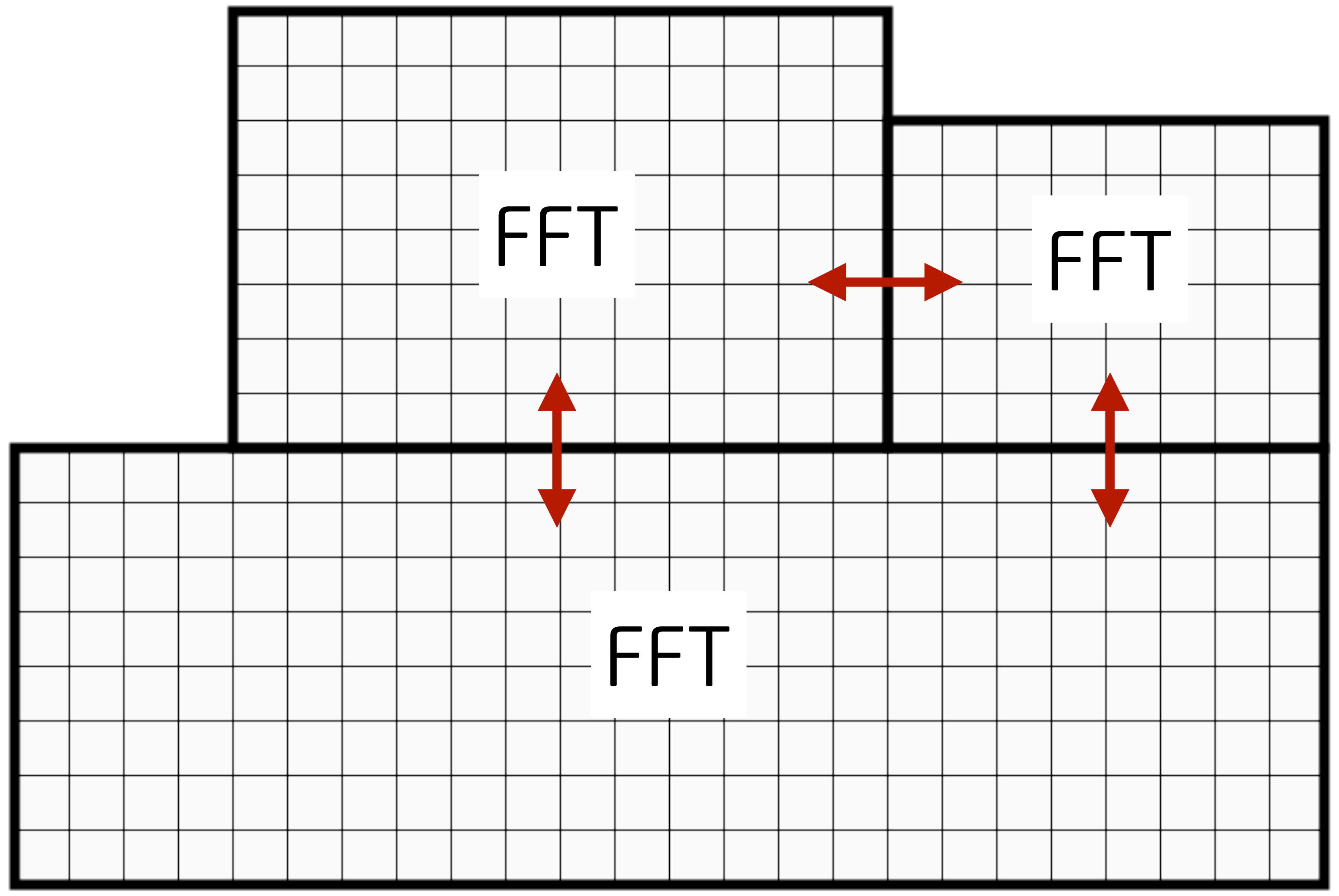
Presentation of different pressure solvers

FFT - FDS Default



Situation in FDS: Rectangular meshes with uniform grid size

Structured



Local data exchanges

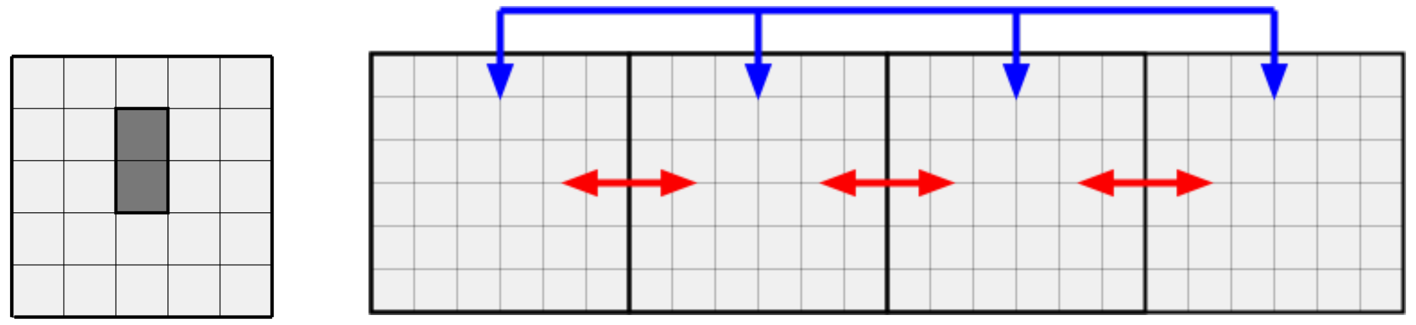
Eigenvectors known on every mesh (sines/cosines)

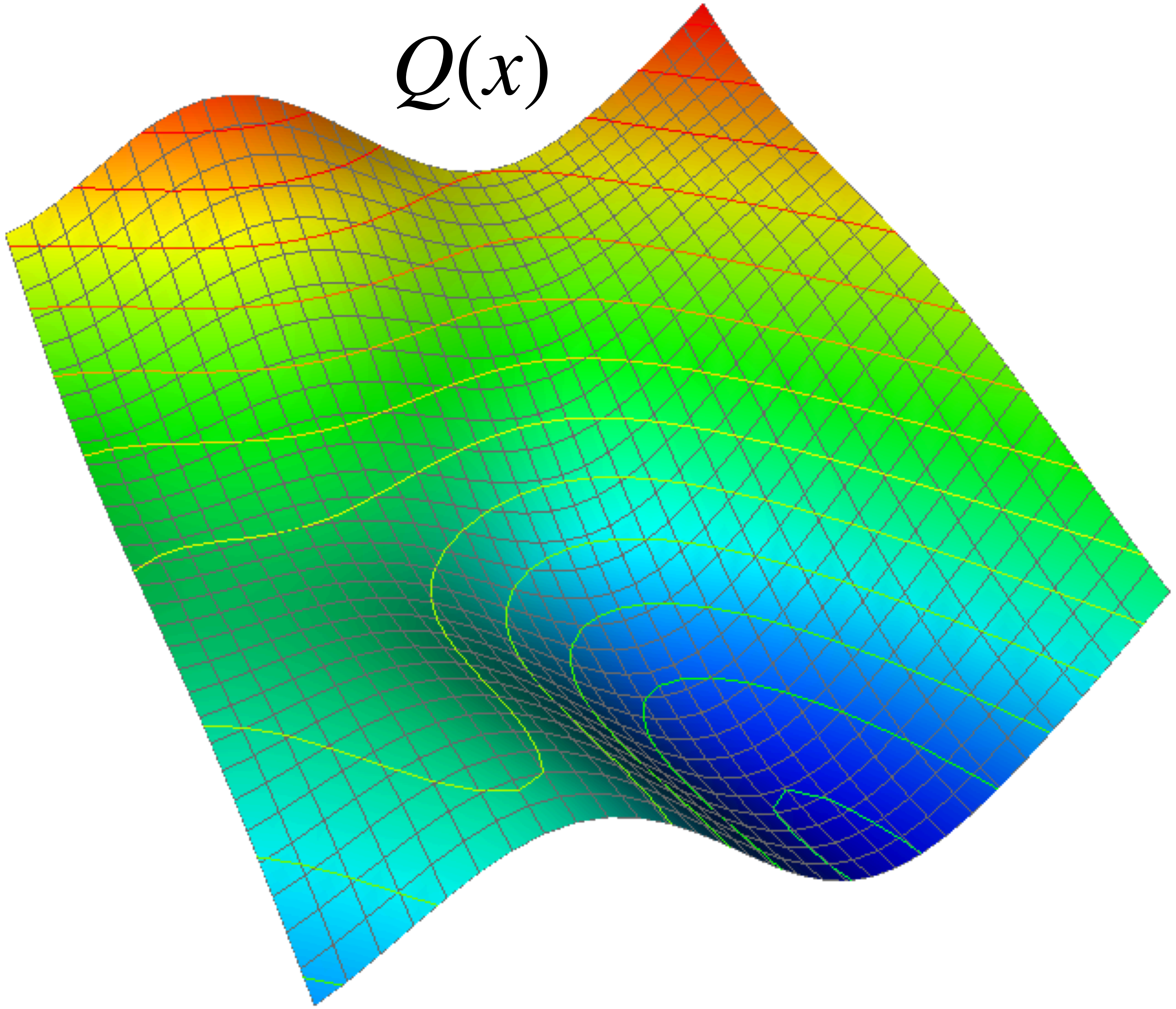
Local Fast Fourier Transformations

Compute local solutions as linear combination of eigenvectors and couple them locally

Highly efficient, but not quite accurate at obstructions and mesh interfaces

Conjugate Gradient

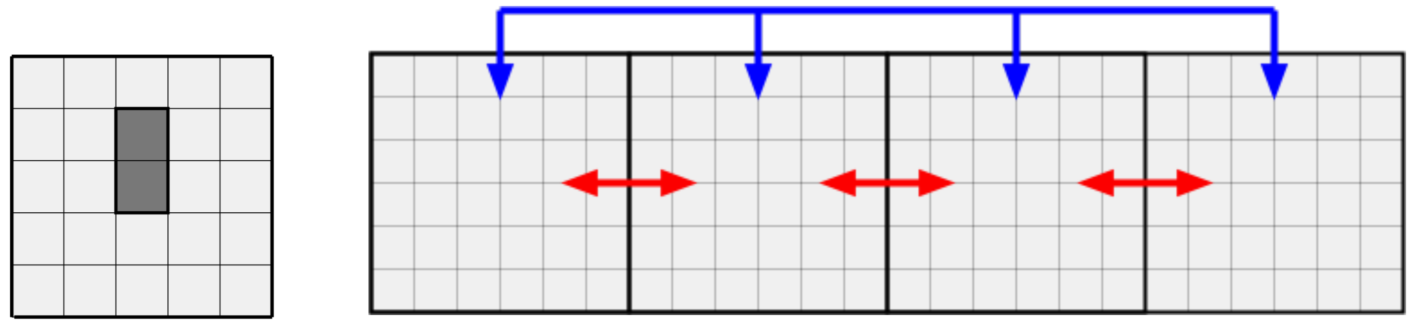


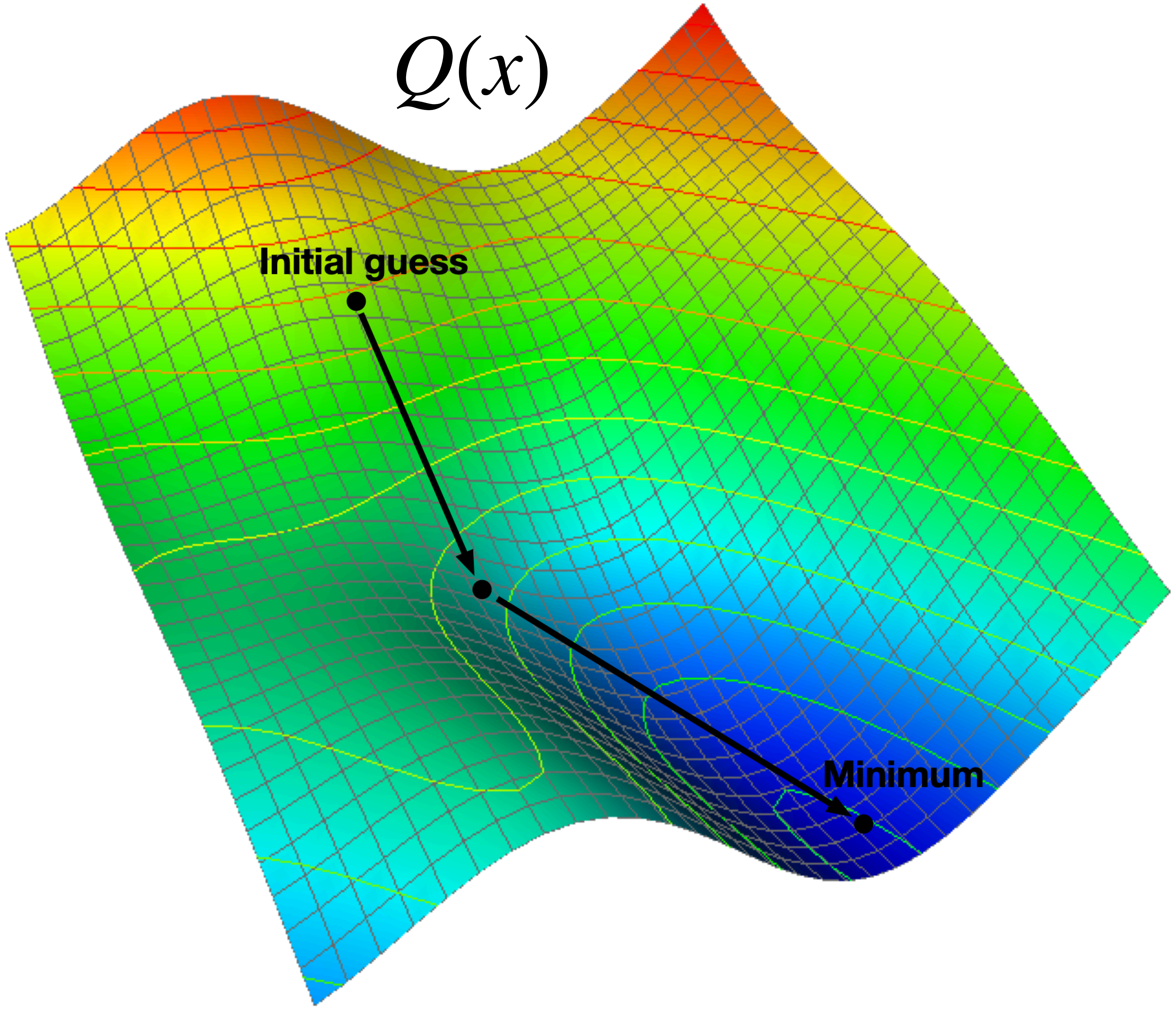


Solution of $Ax = b$ is equivalent to minimum of

$$Q(x) := \left[\frac{1}{2}x^T A x - x^T b\right]$$

Conjugate Gradient





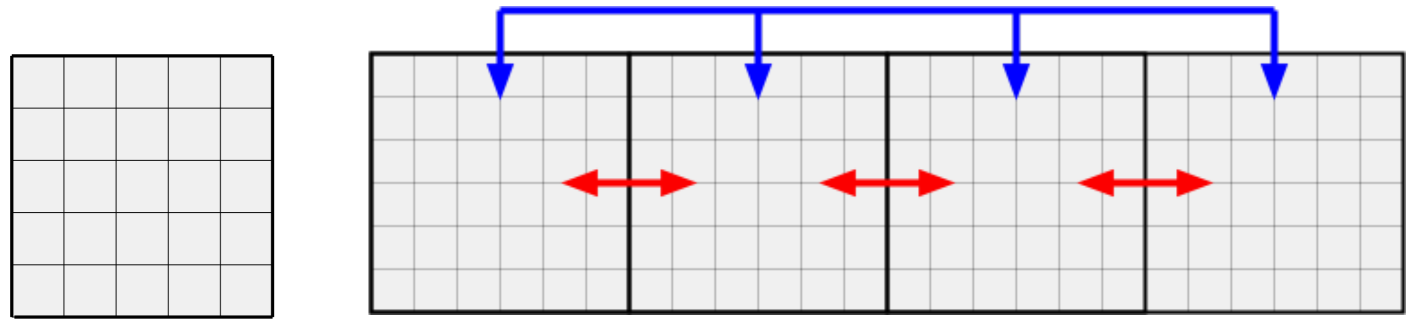
Solution of $Ax = b$ is equivalent to minimum of

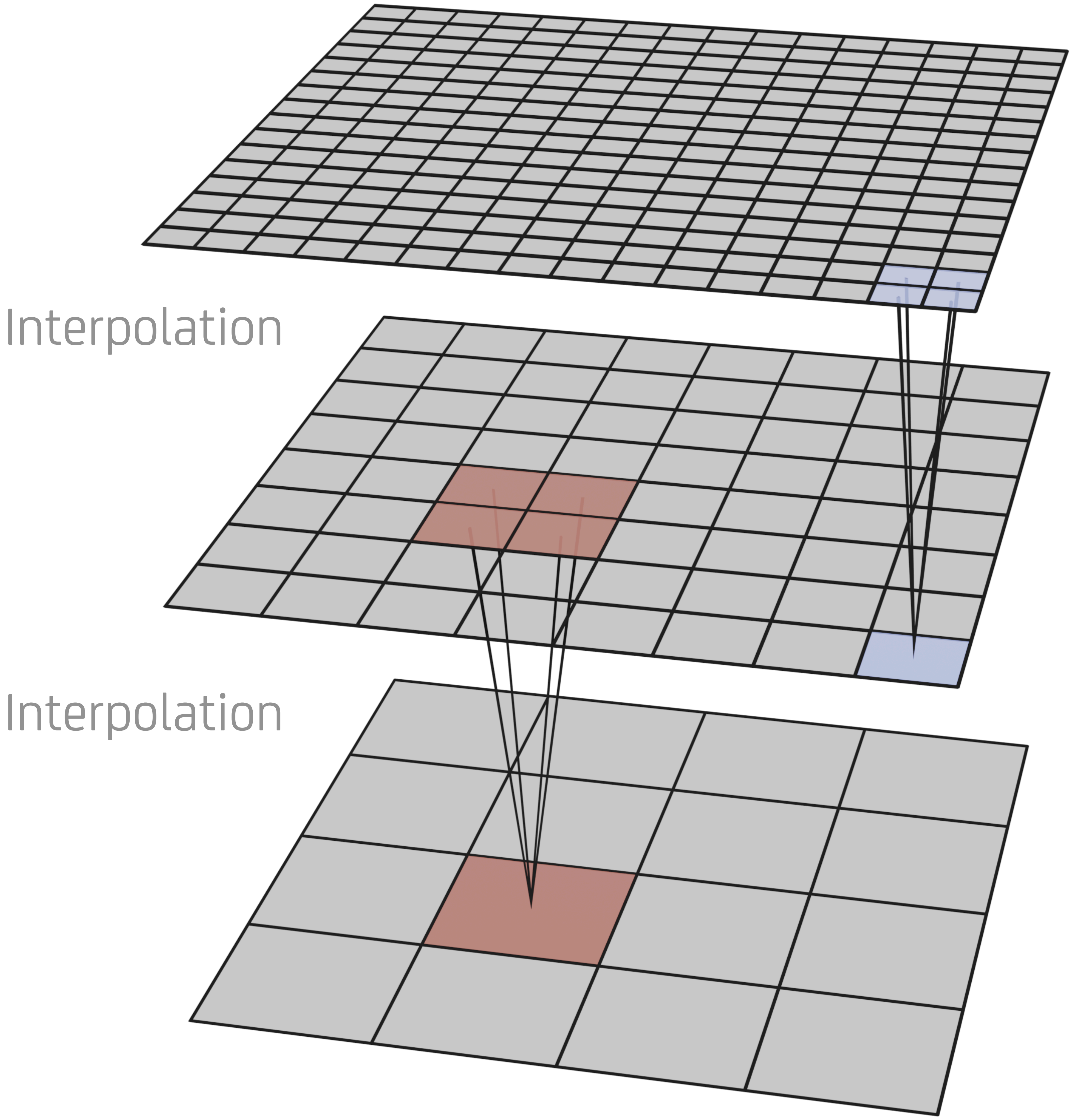
$$Q(x) := \left[\frac{1}{2}x^T A x - x^T b\right]$$

Build sequence of iterates to find minimum of $Q(x)$

Accurate and robust, but slow due to many iterations and global exchanges

Geometric Multigrid





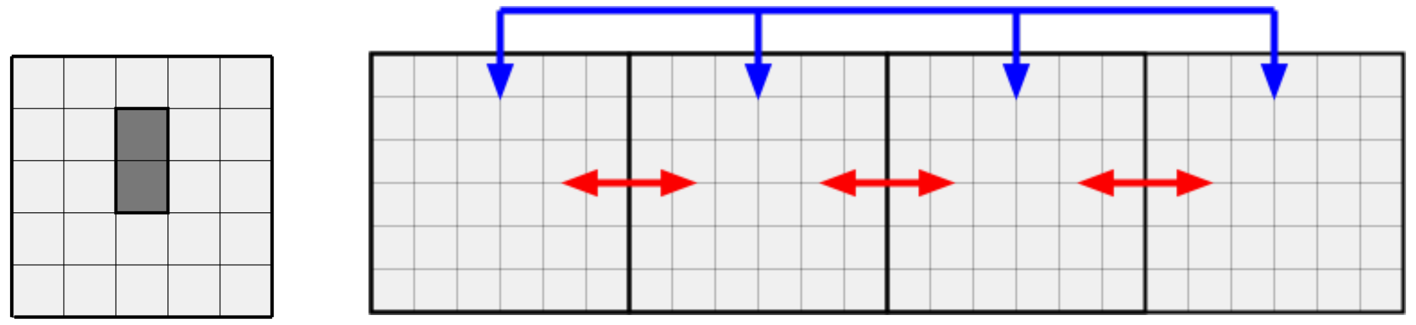
Hierarchy of grids to better map local and global effects

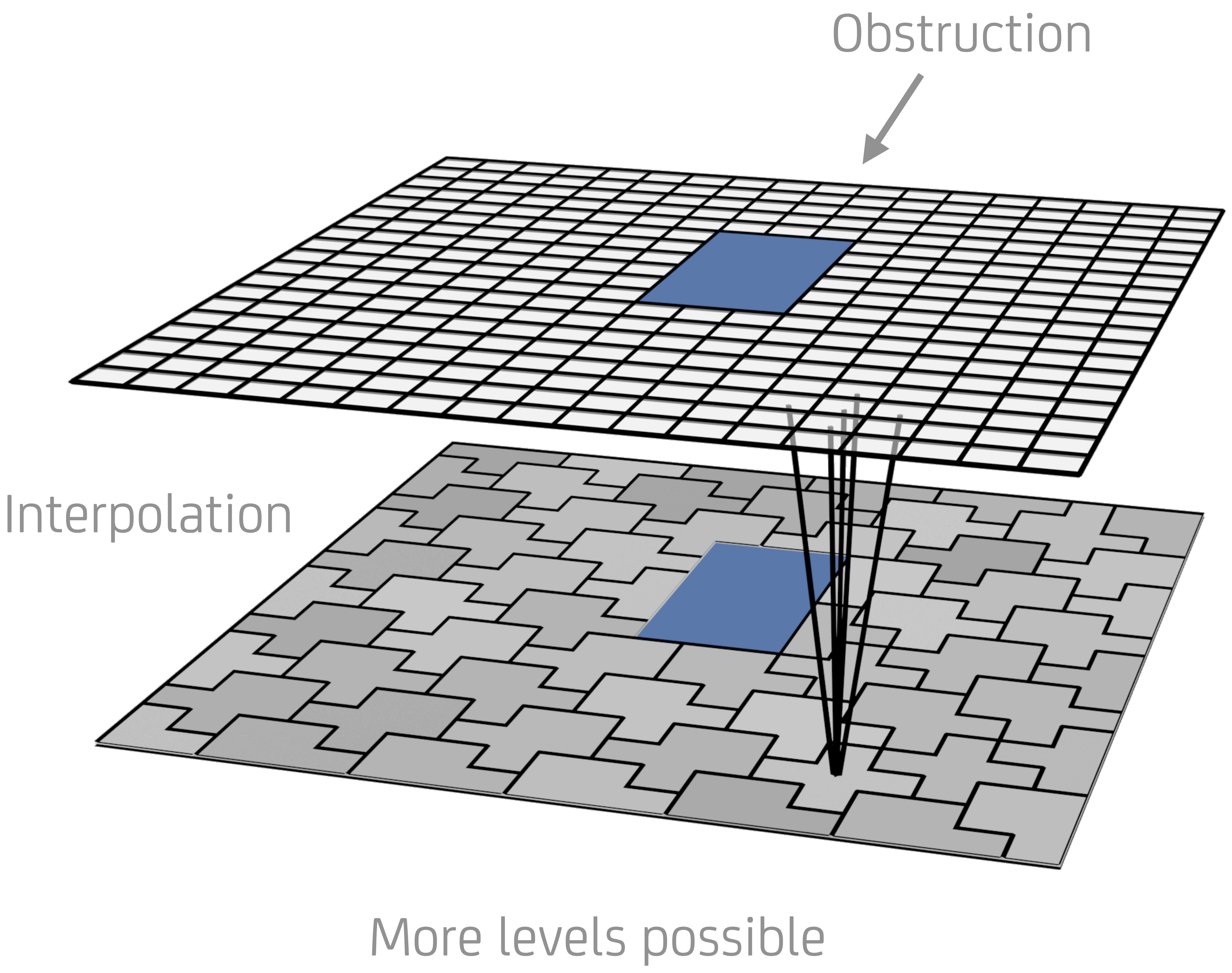
Coarsening based on doubling of mesh width
(restricted to even cell numbers!)

Small obstructions may not fit into coarser grids

Accurate and small number of iterations,
but expensive global data exchanges

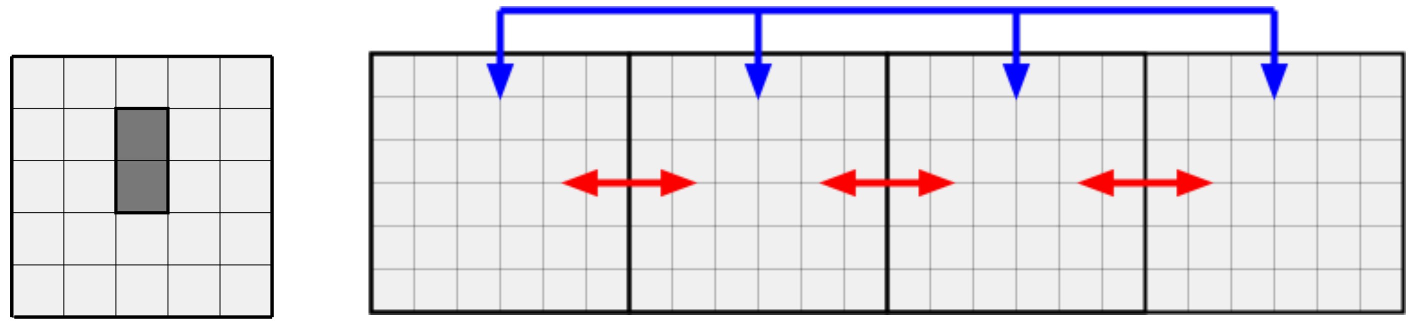
Algebraic Multigrid

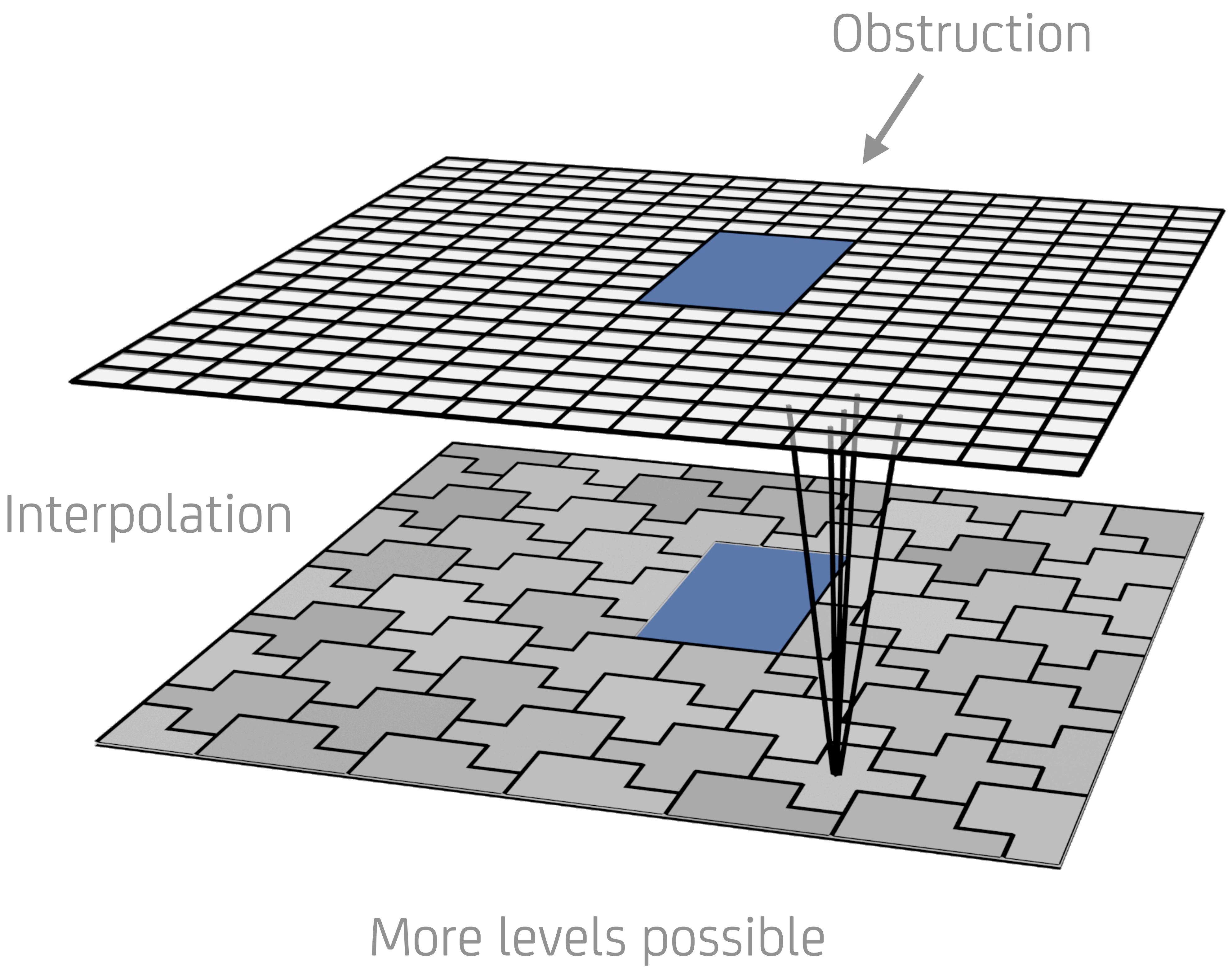




Combine cells into clusters reflecting matrix stencil

Algebraic Multigrid



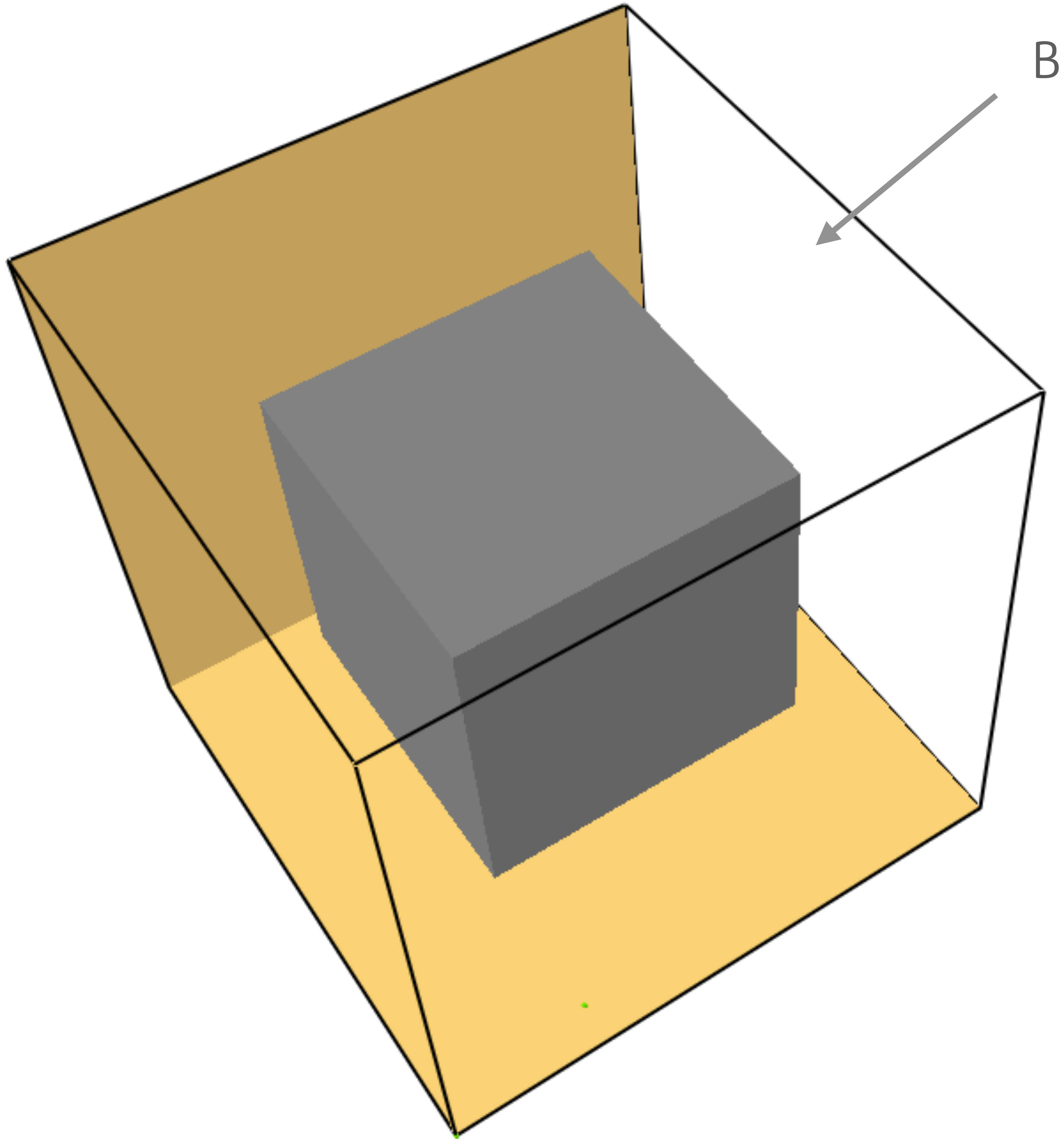


Combine cells into clusters reflecting matrix stencil

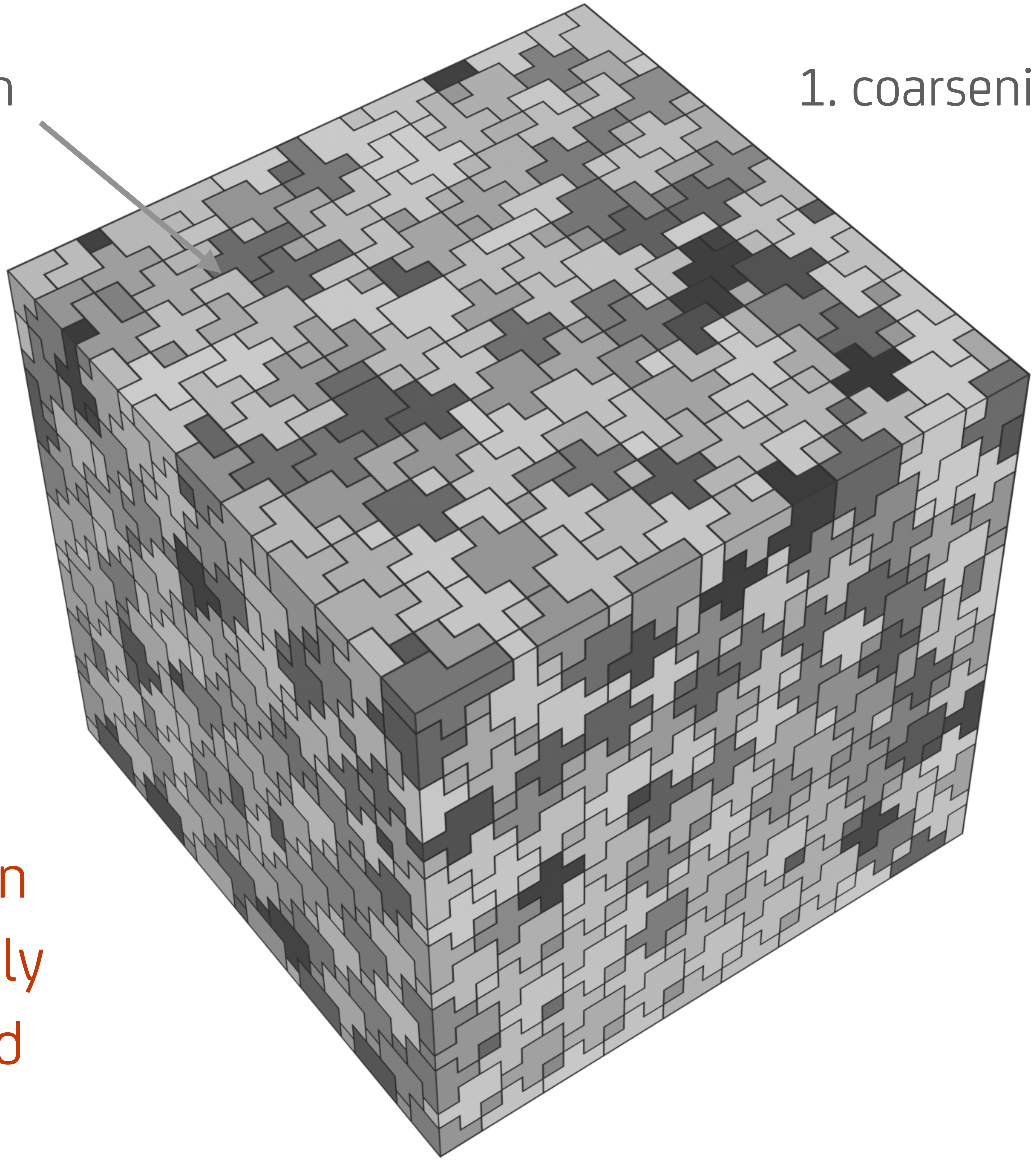
More flexible because arbitrary cell numbers and obstructions can be mapped

Accurate, very small number of iterations, but slow due to global data exchanges

Algebraic multigrid - 3D example



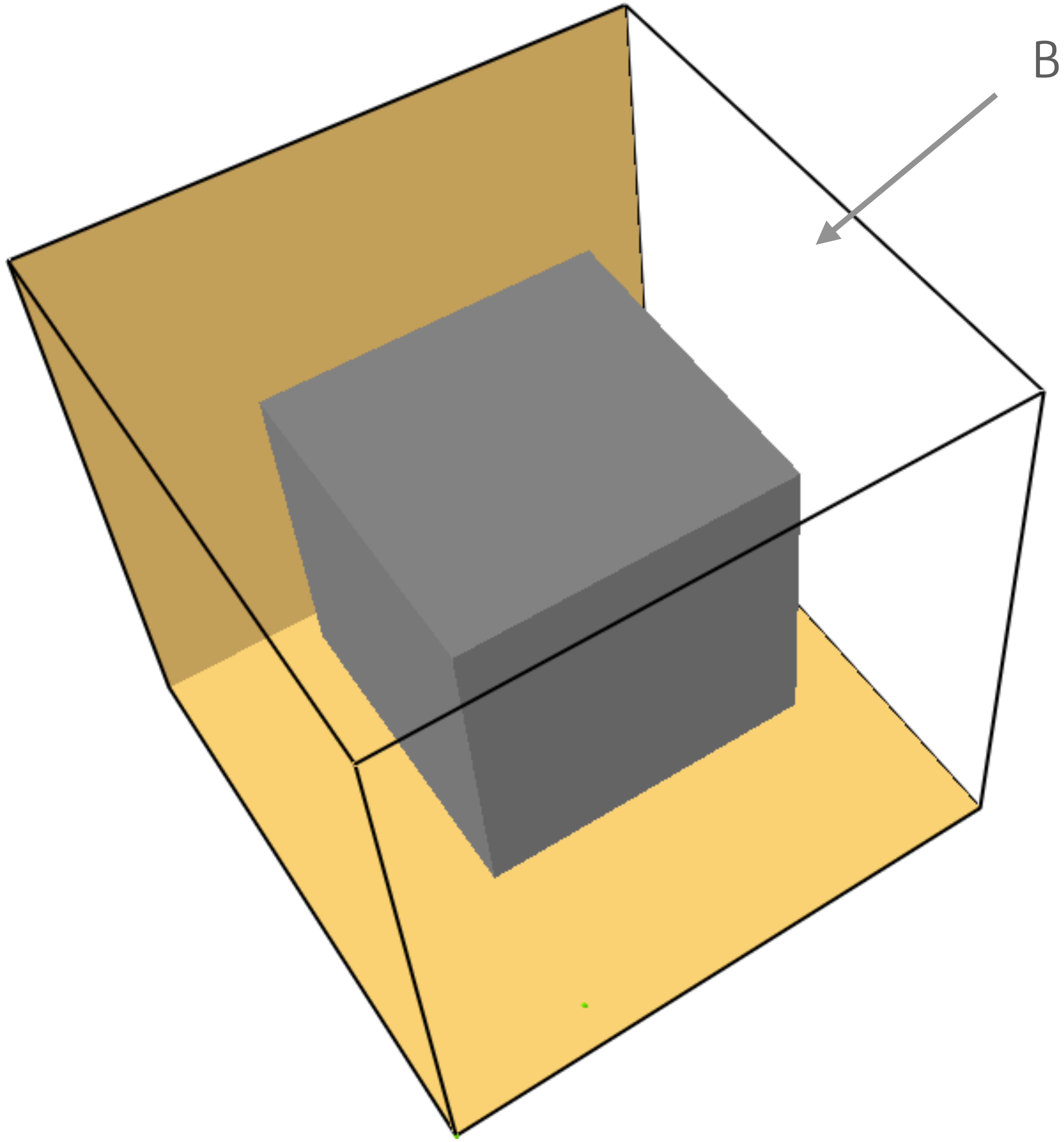
Big obstruction



1. coarsening level

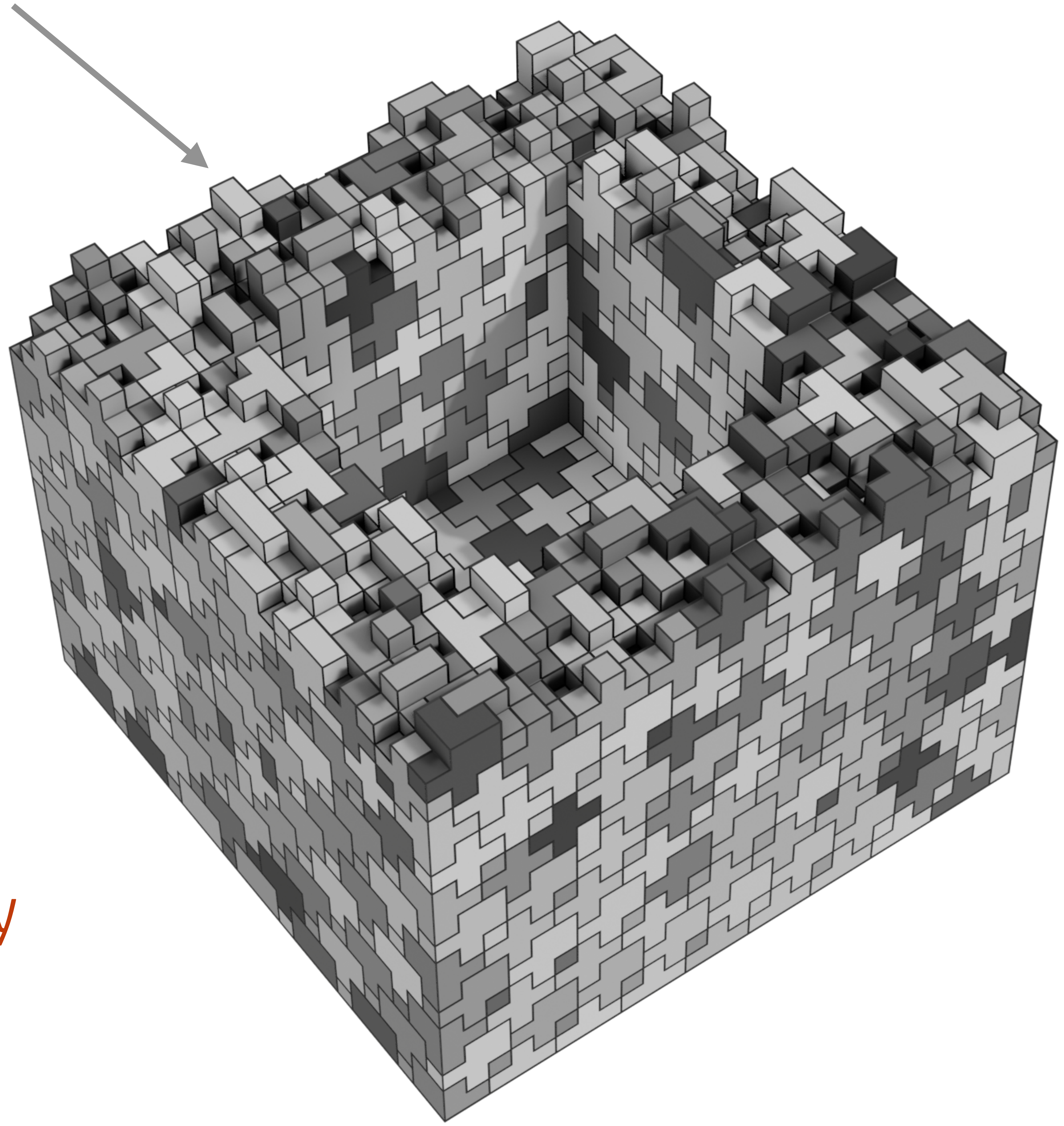
Obstruction
can perfectly
be mapped

Algebraic multigrid - 3D example



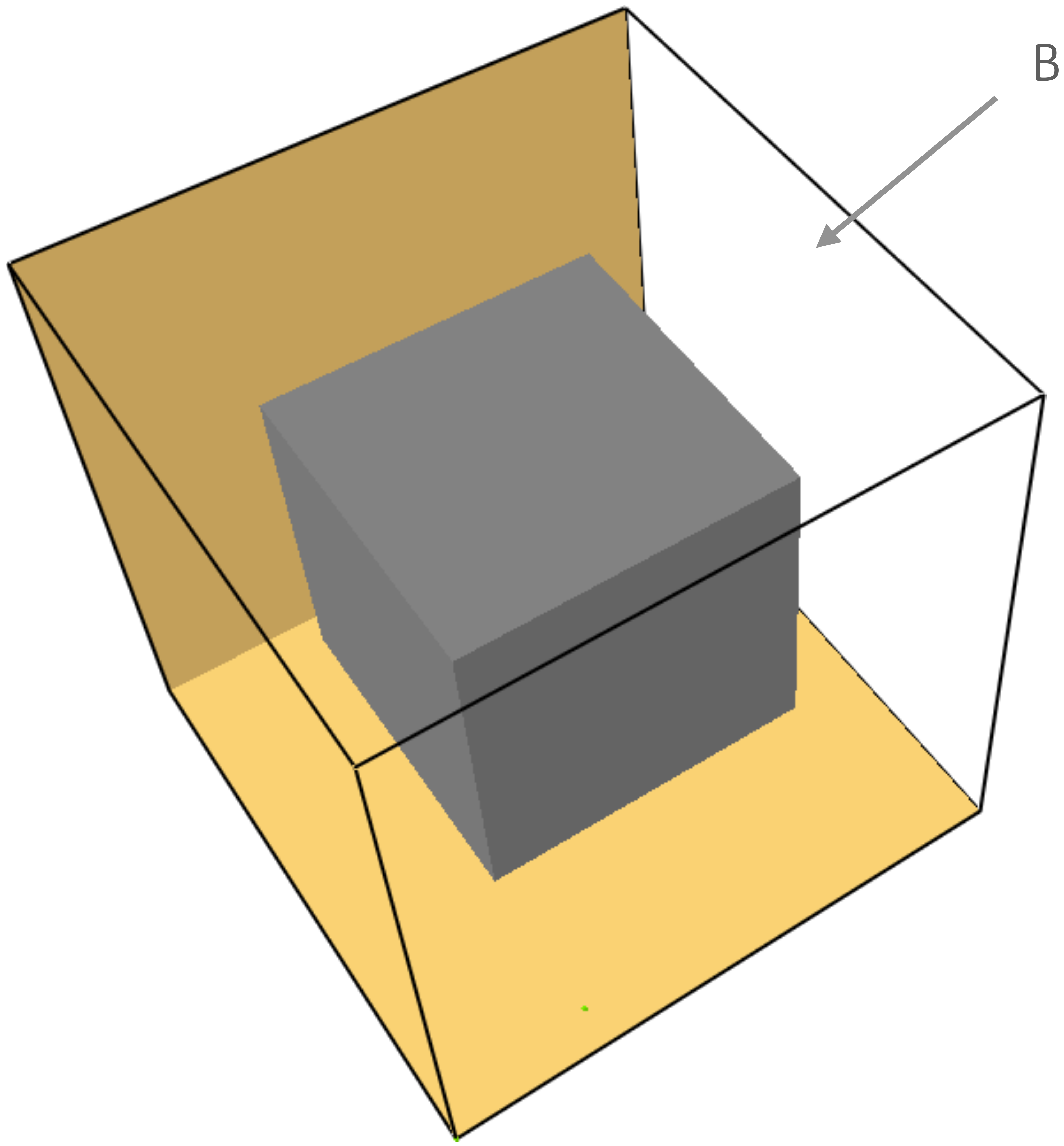
Big obstruction

1. coarsening level



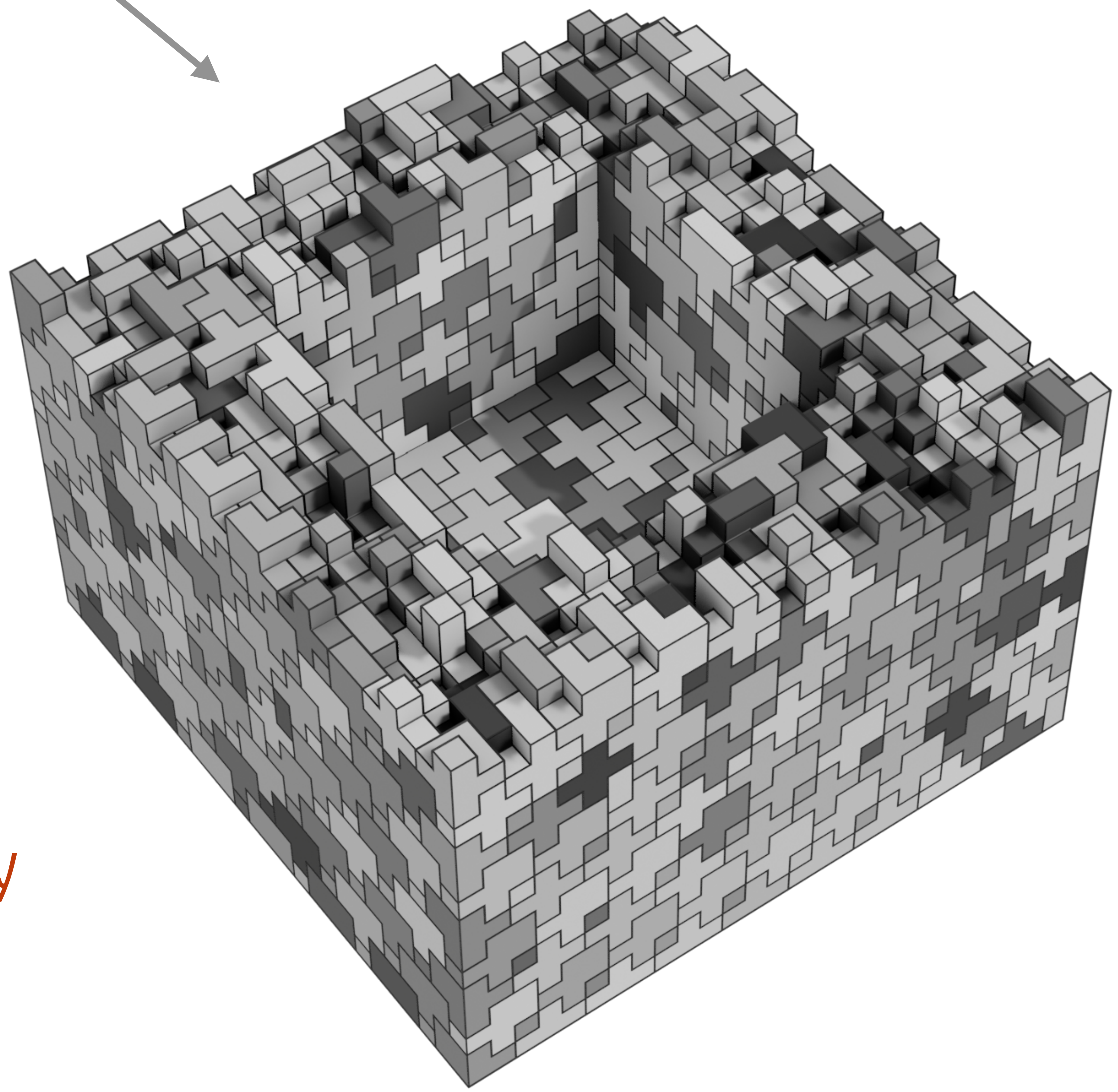
Obstruction can perfectly be mapped

Algebraic multigrid - 3D example



Big obstruction

1. coarsening level

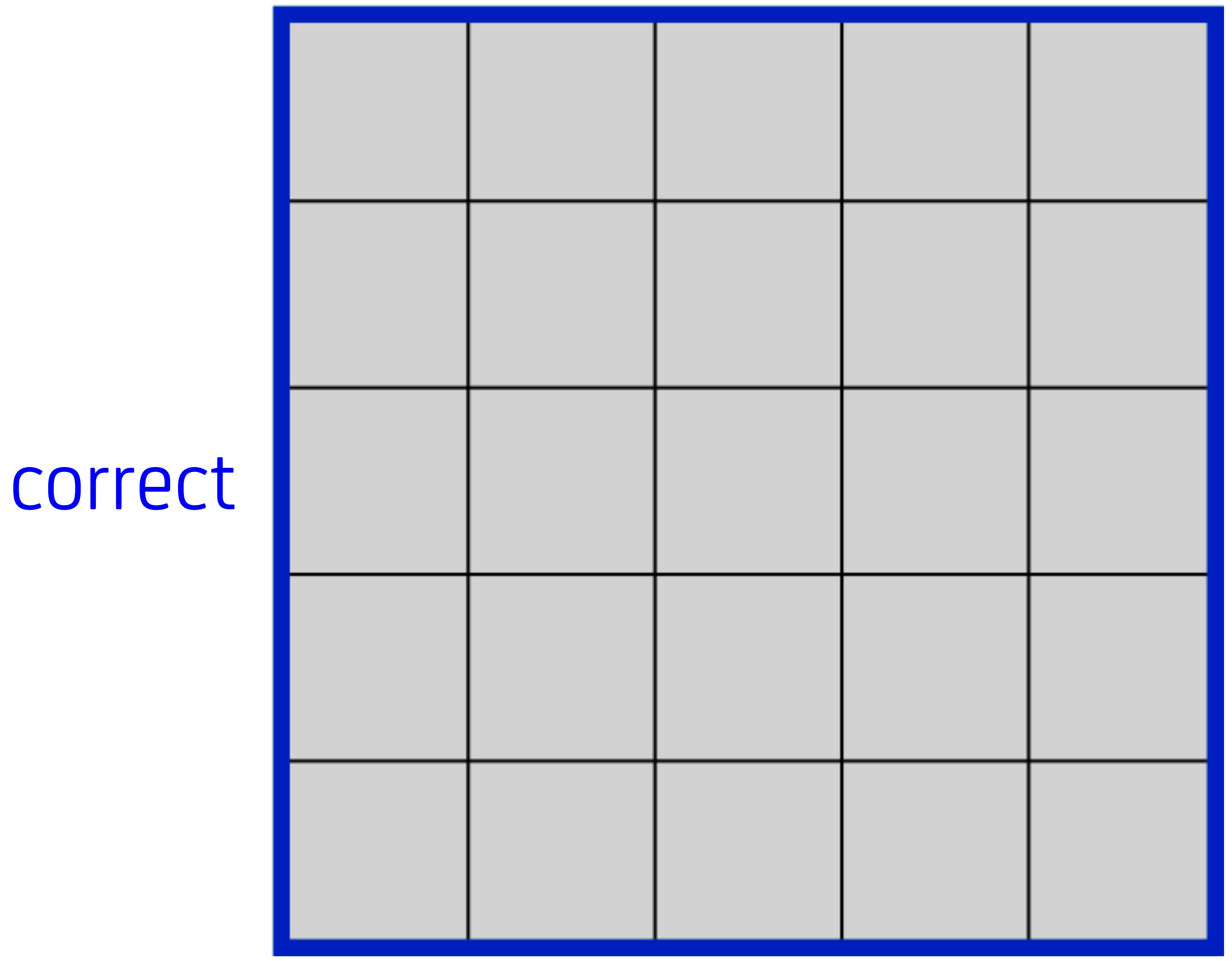


Obstruction
can perfectly
be mapped

McKenney-Greengard-Mayo

Structured problem

correct external boundaries

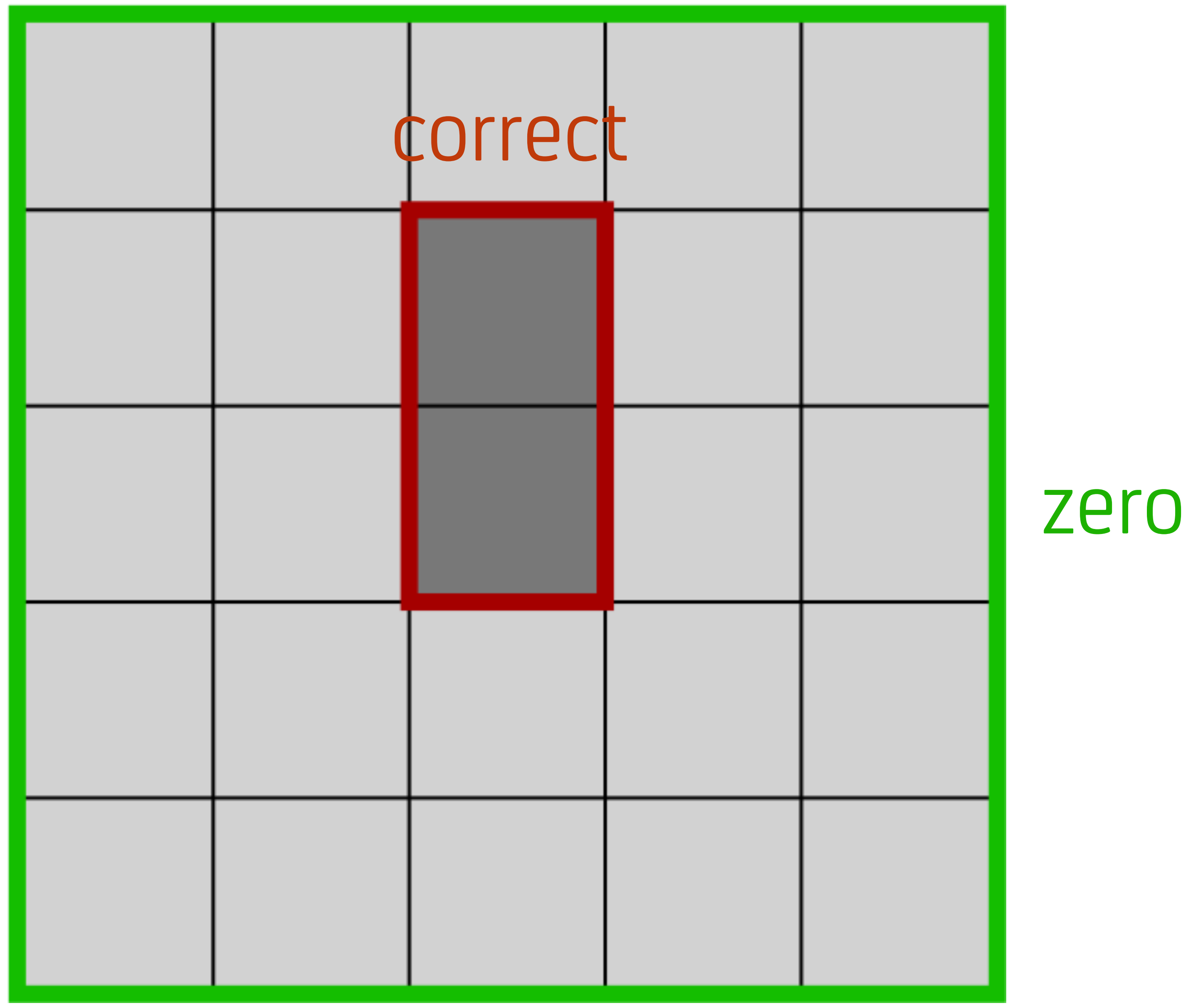


Efficient global structured solver can be applied

+

Unstructured problem

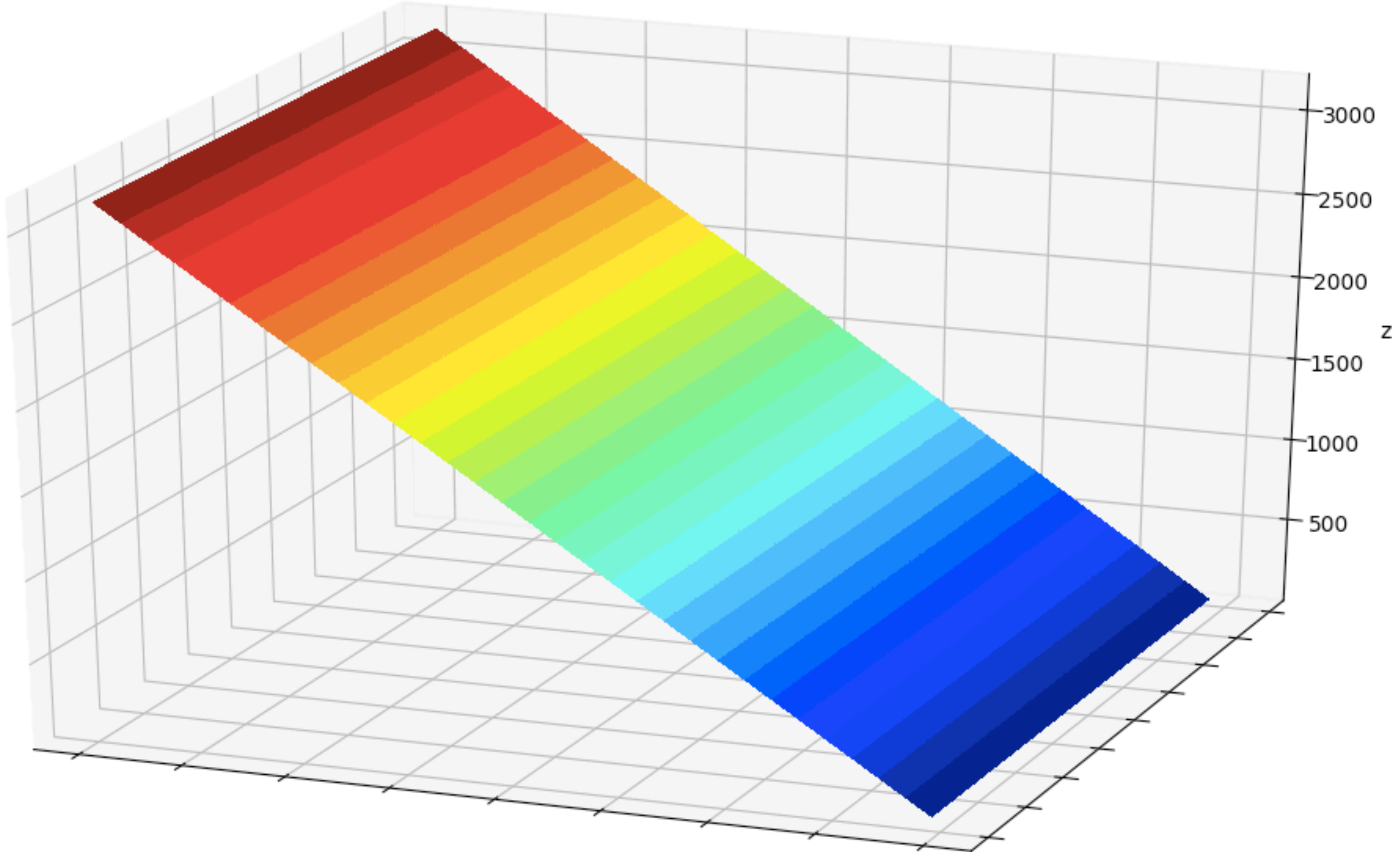
correct internal & zero external boundaries



Many zero entries in RHS, may be exploited

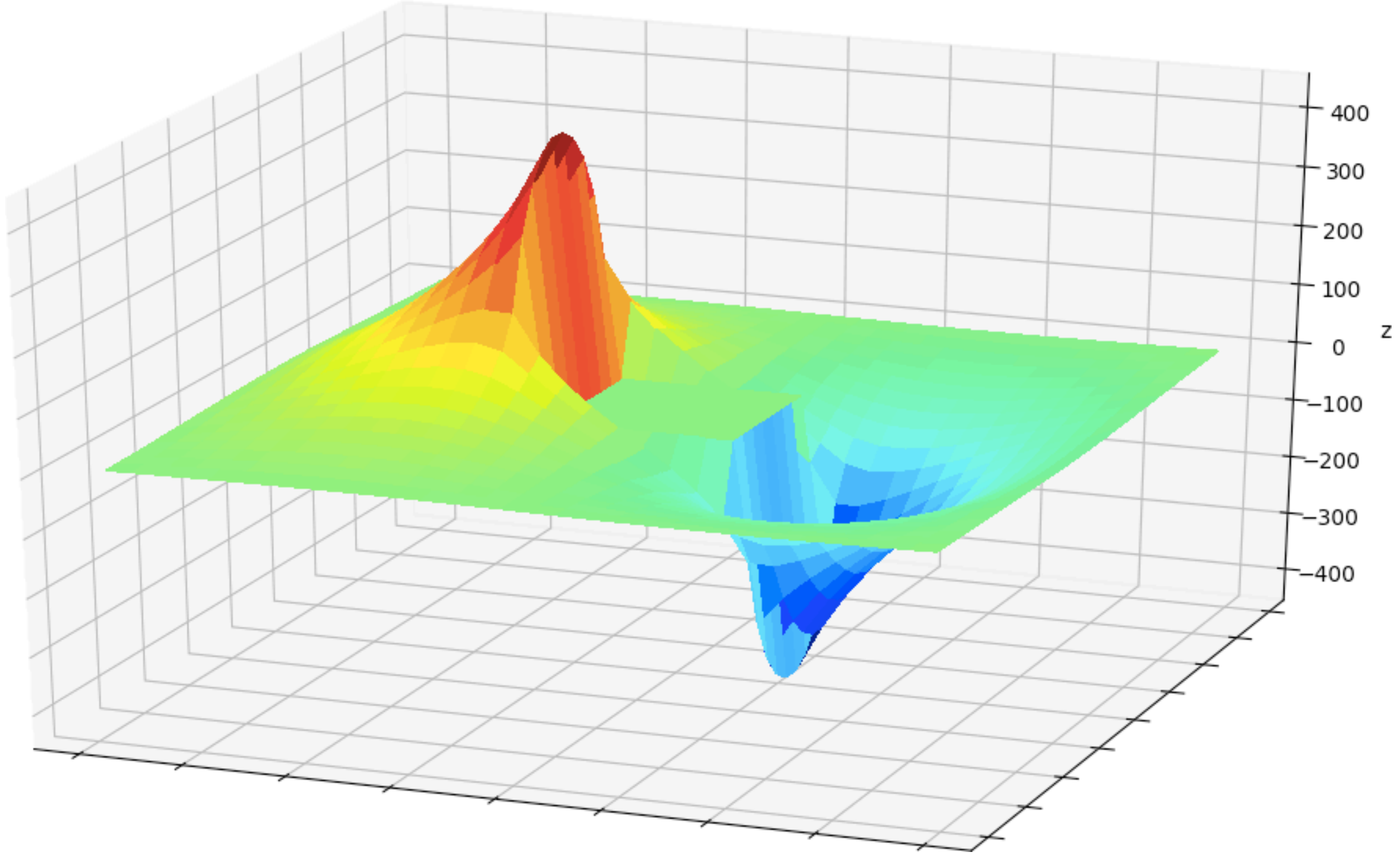
McKenney-Greengard-Mayo

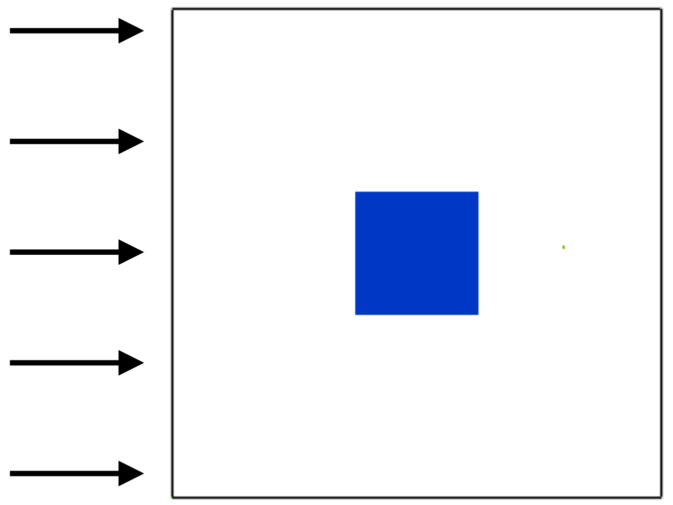
1. Pass:

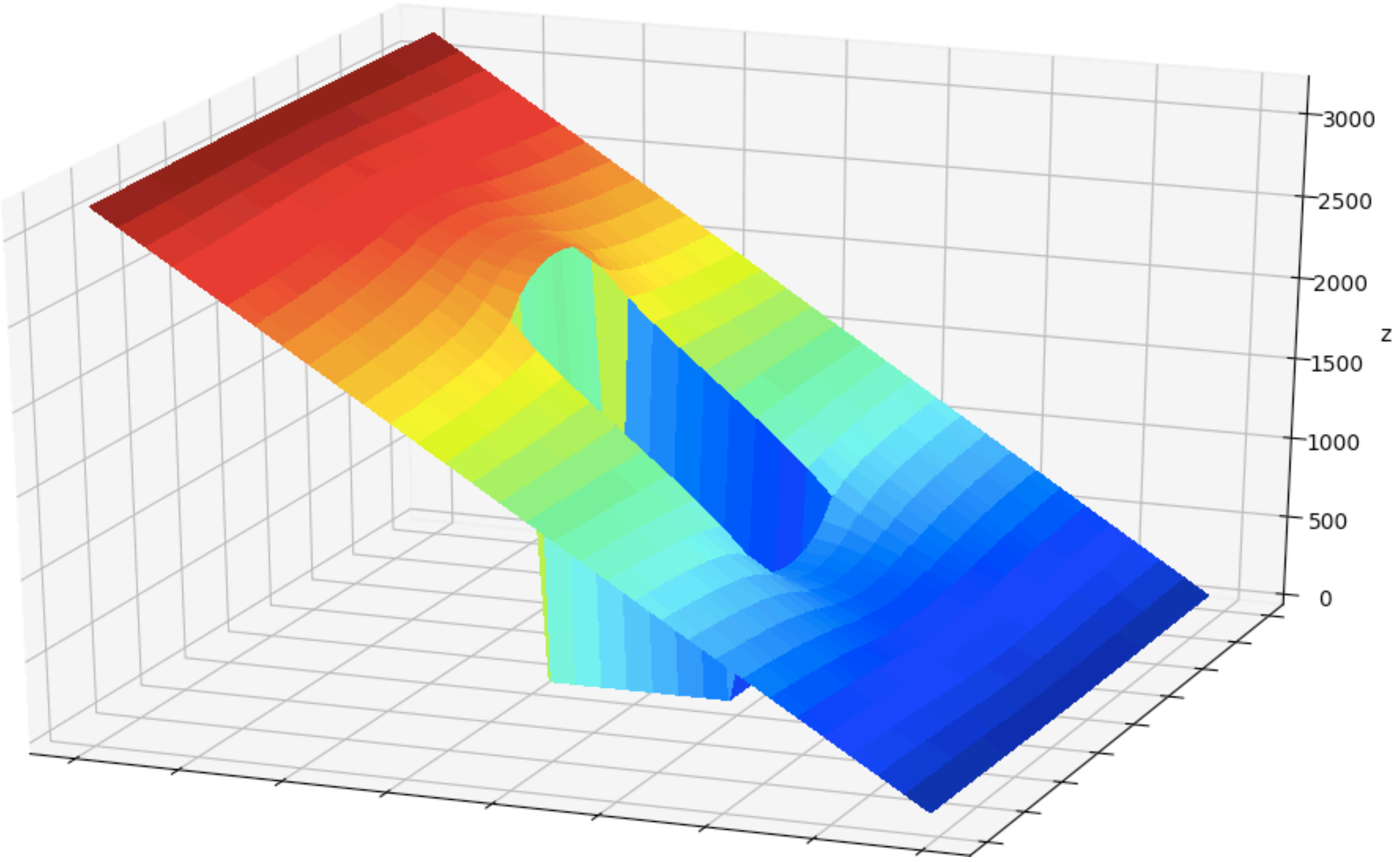


+

2. Pass







In progress,
efficiency still to be analyzed

Thank you very much for your attention